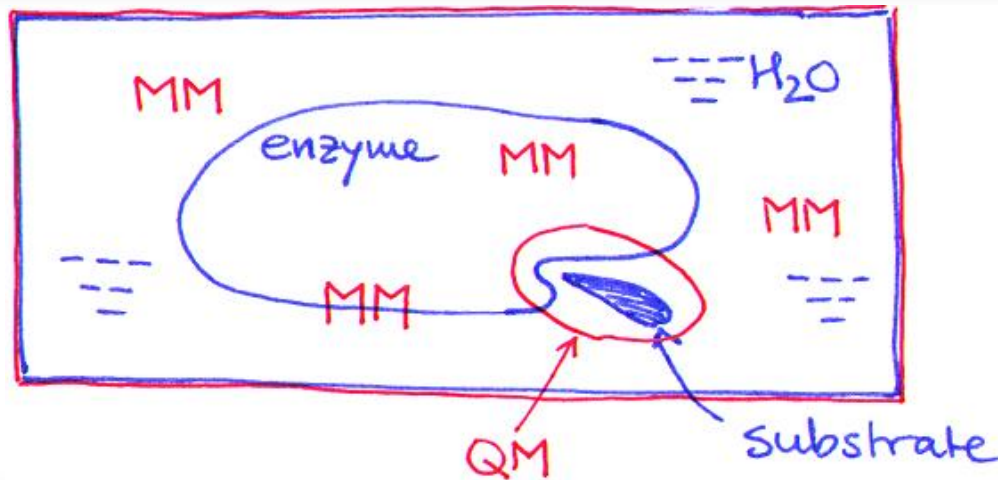


# QM/MM

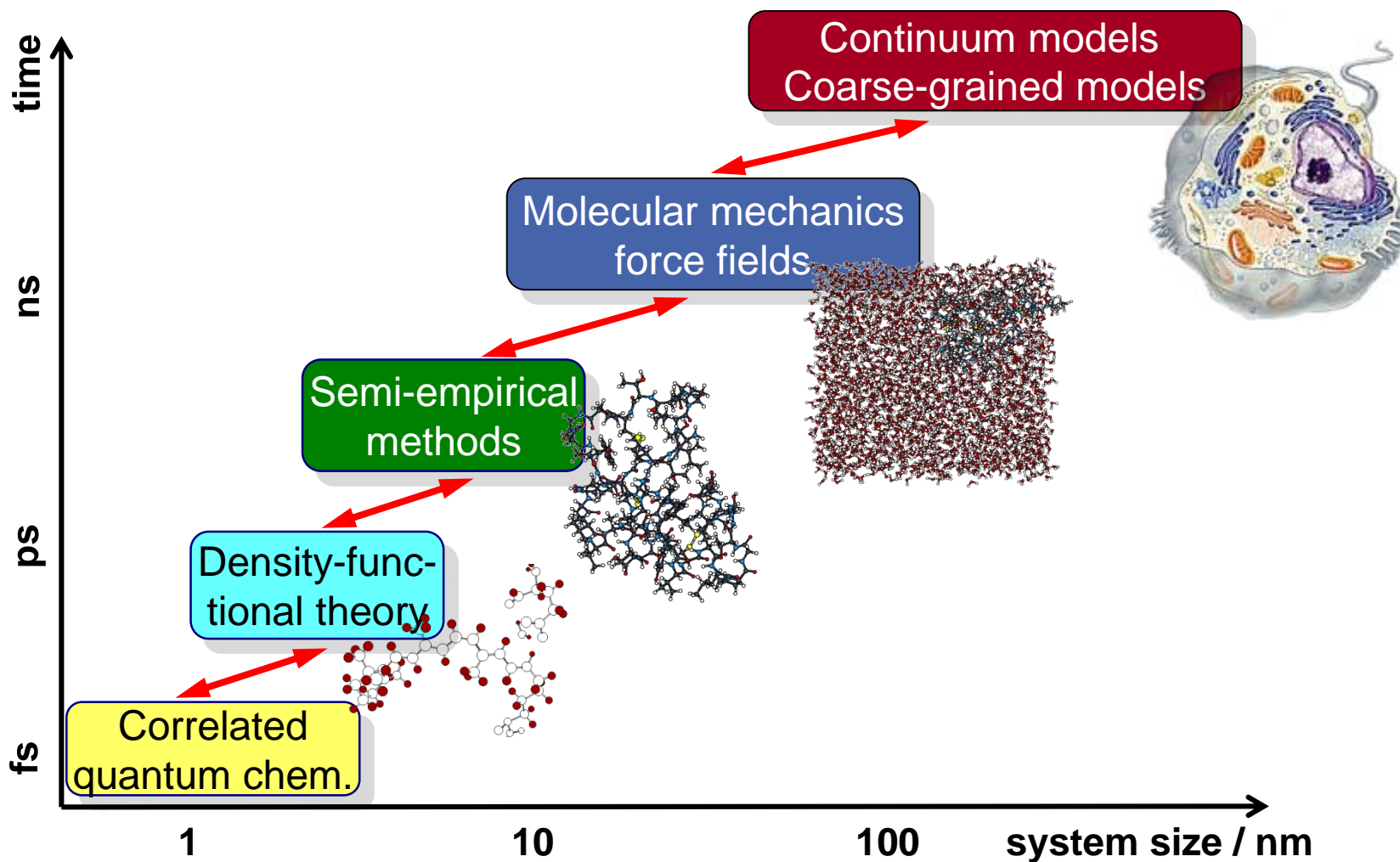
Marcus Elstner and Tomáš Kubař

June 26, 2018

Institute of Physical Chemistry & Center for Functional Nanostructures



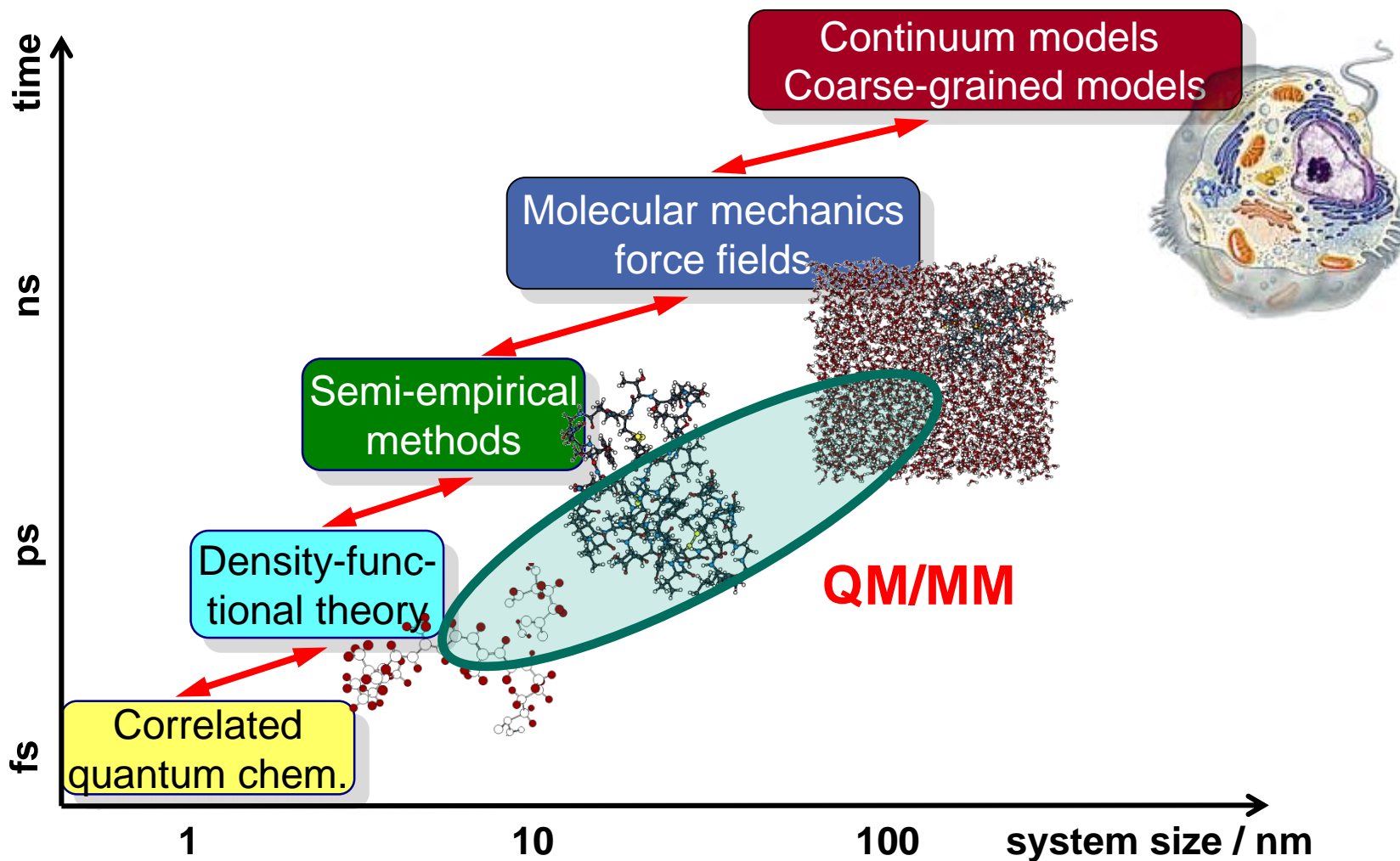
# Computational chemistry



# Computational chemistry

- Molecular mechanics (force fields)
  - Large efficiency
  - Crude approximations necessary
  - Constant electron density → covalent bonds cannot be created / broken
  - Chemical reactions cannot be described
  - Problems also if electron density differs for different conformations
  
- Quantum chemistry
  - No or little assumptions → large transferability
  - Description of chemical reactions possible
  - Larger computational cost
  - **Semi-empirical methods** – involve parameters, compromise (decreased transferability, increased efficiency)

# Computational chemistry



# The Nobel Prize in Chemistry 2013



© Harvard University  
**Martin Karplus**



Photo: © S. Fisch  
**Michael Levitt**

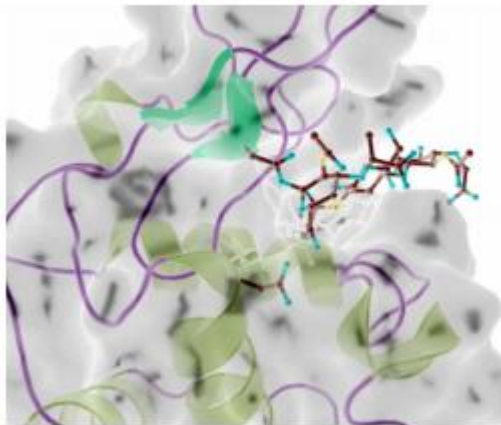


Photo: Wikimedia  
Commons  
**Arieh Warshel**

The Nobel Prize in Chemistry 2013 was awarded jointly to **Martin Karplus**, **Michael Levitt** and **Arieh Warshel** *"for the development of multiscale models for complex chemical systems"*.

# Hybrid QM/MM methods

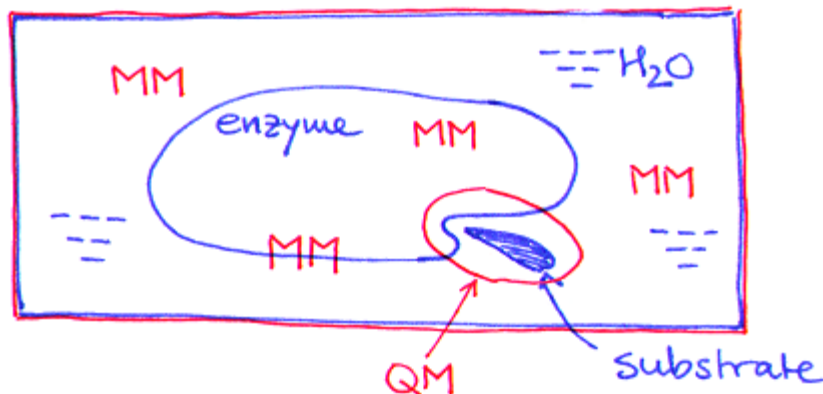
- Assumption – changing electron density localized in a small part of the molecular system  
– e.g. the binding site of a ligand in an enzyme



- Only a small number of atoms are involved in a chemical reaction
- The large remainder of the system stays outside of the process, but still may interact by way of non-covalent interactions

# Hybrid QM/MM methods

- Assumption – changing electron density localized in a small part of the molecular system  
– e.g. the binding site of a ligand in an enzyme



- Region where chemical reaction occurs – quantum chemistry (QM)
- The large remainder – MM force field
- Interaction of both regions must be accounted for – QM/MM / embedding

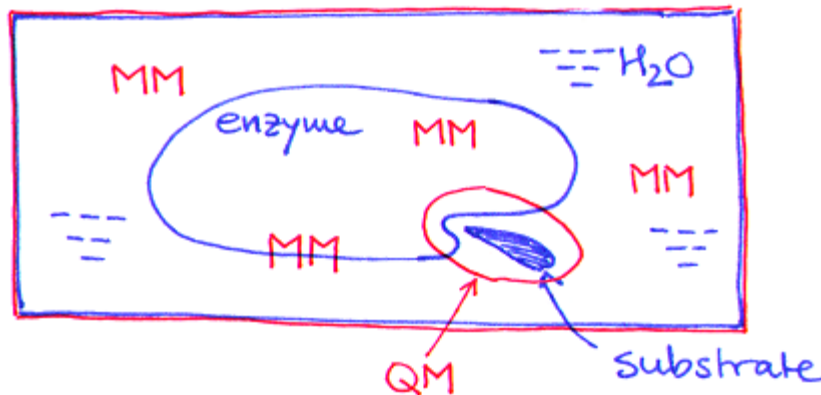
$$E_{\text{total}} = E_{\text{QM}} + E_{\text{MM}} + E_{\text{QM/MM}}$$



# Hybrid QM/MM methods

- Interaction of both regions must be accounted for – QM/MM / embedding

$$E_{\text{total}} = E_{\text{QM}} + E_{\text{MM}} + E_{\text{QM/MM}}$$



- The QM and MM calculations – performed in the usual way
- Key issue – how to couple the QM and MM regions?

Goal – to obtain  $E_{\text{QM/MM}}$



# Embedding schemes

- Many various schemes proposed
- Differences – how much of the QM–MM interaction is neglected

In the order of increasing complexity / completeness:

- Mechanical embedding
- Electronic embedding
- Polarized embedding

# Mechanical embedding

- Unpolarized interactions
- Entirely independent QM and MM calculations
- Interaction between the QM and MM regions – with a force field

$$E_{\text{QM/MM}} = \sum_i^{\text{QM atoms}} \sum_m^{\text{MM atoms}} \left( \frac{q_i^{\text{Mull}} \cdot q_m}{r_{im}} + 4\epsilon_{im} \left( \frac{\sigma_{im}^{12}}{r_{im}^{12}} - \frac{\sigma_{im}^6}{r_{im}^6} \right) \right)$$

- QM atoms have to be assigned Lennard-Jones parameters and their quantum-chemical charges are used (e.g. Mulliken charges)
- Specific combinations of force fields and quantum-chemical methods lead to very good results for specific classes of molecules / reactions
- Generally, care must be taken

# Mechanical embedding

- Problem – the electron density in the QM region is unaffected by ('does not feel') the MM region – therefore 'unpolarized' scheme
- As if the QM region was calculated 'in vacuum'
  - this may deviate from reality considerably
  - imagine that the reaction center of the enzyme is surrounded by charged amino acid side chains – affects the reaction strongly

# Electronic embedding

- Polarized QM / unpolarized MM
- Electrostatic interaction with the MM atomic charges is included in the QM Hamiltonian
- Works with any electronic structure method
  - HF, DFT, correlated wave-function, semi-empirical...

# Electronic embedding

Interaction of QM electrons with MM point charges

- moves from the QM/MM term (described with a force field) to the quantum chemical contribution
- the charge density interacts with a set of point charges
- *little* increase of the computational cost

$$\hat{H}'_{\text{QM}} = \hat{H}_{\text{QM}} - \sum_j^{\text{QM electrons}} \sum_m^{\text{MM atoms}} \frac{q_m}{r_{jm}}$$

Interaction of QM nuclei with MM point charges may remain in QM/MM

$$E'_{\text{QM/MM}} = \sum_i^{\text{QM atoms}} \sum_m^{\text{MM atoms}} \left( \frac{Z_i \cdot q_m}{r_{im}} + 4\epsilon_{im} \left( \frac{\sigma_{im}^{12}}{r_{im}^{12}} - \frac{\sigma_{im}^6}{r_{im}^6} \right) \right)$$

# Electronic embedding

- A good approach with chances for acceptable accuracy
- Possible imbalance
  - QM region is polarized by MM
  - MM region cannot be polarized  
because of the constant atomic charges

# Polarized embedding

- Fully polarized interactions
- Involves a polarizable MM force field
  - Every MM atom is assigned a polarizability  $\alpha$
  - Induced point dipole on the atom obtained from the field intensity  $\vec{E}$ :
$$\vec{\mu}^{\text{ind}} = \alpha \cdot \vec{E}$$
  - To be calculated iteratively until self-consistence
  - Possible problems – computational cost, difficult convergence of dipoles
- The induced dipoles interact with MM point charges as well as with the QM electron density and nuclei
- Both QM and MM calculations are iterative  
→ convergence of both is required!



# Polarized embedding

- Formally the most complete approach
- Dramatically increased computational cost
- Question: Does the improvement of the quality of results justify the extreme cost of calculation?
- Electronic embedding may be sufficient in many cases

# QM/MM study – work plan

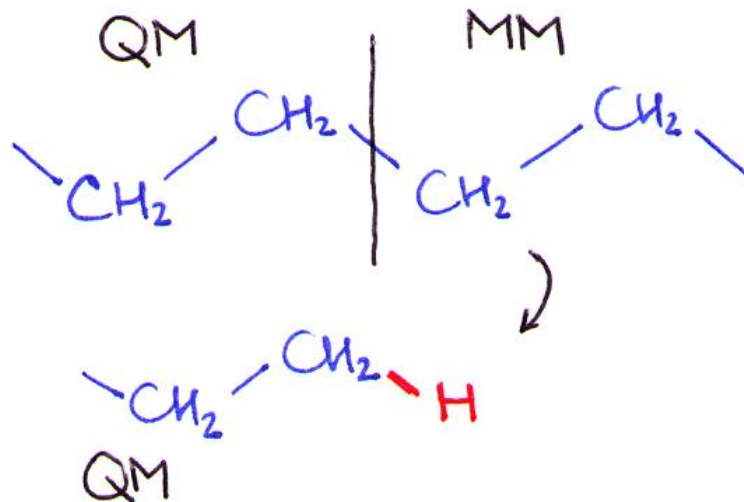
1. Define the QM region
  - shall involve all of the interactions for which MM is insufficient
2. Choose the QM (and MM) methods
  - QM is the crucial and by far most consuming part of calculation
  - accuracy as well as efficiency shall be judged
3. Determine the type of embedding (perhaps electronic)
  - also introduce LJ parameters for QM atoms (usual MM, or special)
4. Simulate
  - each MD step involves 1 QM calculation (and 1 MM, 1 QM/MM)
5. Analyze

# Covalent bond crossing the QM/MM boundary

- What shall be done if a QM atom and an MM atom are bound covalently?
- Usually inevitable in studies of proteins...
- Special treatment of the boundary is necessary  
(because the QM region is no real 'molecule' etc.)
- Linear combination of molecular fragments
- Link atoms (additive or subtractive coupling)
- Frozen orbitals
- other, further, current development...

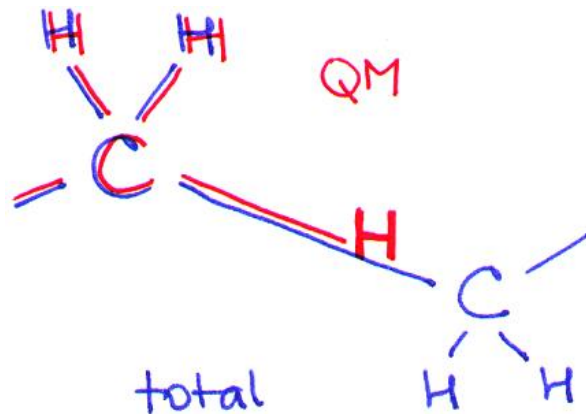
# Link atom approach

- We need a description suitable even if there is QM–MM electrostatics
- Typically in proteins – polar and charged amino acids in the MM region
  - polarization of the electron density of the QM region
- Missing in the electronic embedding
  - description of covalent bonds crossing the QM–MM boundary
- **Link atom** – replaces the covalently bonded MM system at the boundary



# Link atom approach

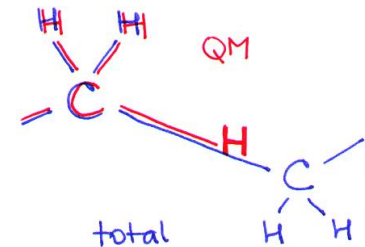
- Usually:
  - bonds between two  $sp^3$ -carbon atoms chosen to be cut
  - hydrogens used as link atoms (similar electronegativity of C and H)
    - also methyl ‘atoms’ may be used with some quantum chem. methods
- Desirable – define the QM region such that
  - the bonds to be cut are as unpolar as possible (to minimize errors)
- Link atom – placed on the original C–C bond in a typical C–H distance



# Link atom approach

## Additive coupling

- Bonded interactions crossing the QM–MM boundary calculated with the MM force field
- Sometimes omitted – angles involving 2 QM atoms and 1 MM atom, dihedrals with 3 QM atoms and 1 MM atom
- MM atoms situated close to the QM system
  - almost overlap with the link atom
  - unphysically large influence on the electronic structure of the QM system
  - their charges may be scaled down or even removed
  - good alternative 1: replace point charges by gaussian charge distributions
  - good alternative 2: divide the charge among not-so-close MM atoms  
(motivation: the total charge should remain, and should not move far)



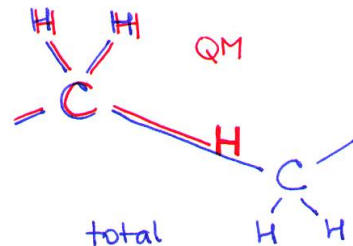
# Link atom approach

## Subtractive coupling

- Two MM calculations: one for the entire system, and one for the QM region including the link atom(s) – the same as for the QM calculation

$$E_{\text{total}} = E_{\text{MM}}^{\text{large}} + (E_{\text{QM}}^{\text{small}+\text{L}} - E_{\text{MM}}^{\text{small}+\text{L}})$$

(large = entire system, small+L = QM region)



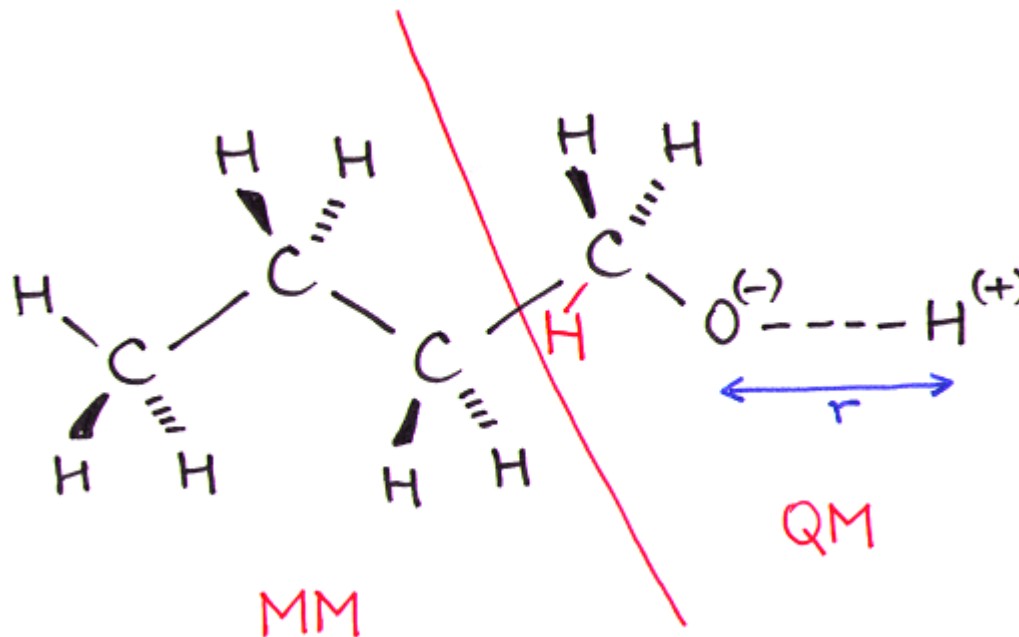
- May be deficient in some applications...



# Link atom approach

Deficiency of subtractive coupling – example

- Reaction  $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{OH} \rightarrow \text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{O}^- + \text{H}^+$
- QM region – methanol fragment, MM region – remainder
- Goal – evaluate the energy as a function of the O–H distance  $r$



# Link atom approach

Deficiency of subtractive coupling – example

■ Reaction



■ Assume – parameters for methanol and butanol differ only in the force constant  $k$  for the O–H bond;  
all remaining terms give a constant independent of  $r$

$$\begin{aligned} E_{\text{butanol}}^{\text{QM/MM}}(r) &= E_{\text{methanol}}^{\text{QM}}(r) + \left( \frac{1}{2} k_{\text{butanol}}^{\text{OH}} \cdot (r - r_0)^2 - \frac{1}{2} k_{\text{methanol}}^{\text{OH}} \cdot (r - r_0)^2 \right) + \text{const.} \\ &= E_{\text{methanol}}^{\text{QM}}(r) + \frac{1}{2} (k_{\text{butanol}}^{\text{OH}} - k_{\text{methanol}}^{\text{OH}}) \cdot (r - r_0)^2 + \text{const.} \end{aligned}$$

■ MM energy – retains the form of  $r^2$

■ QM energy – for large  $r$ , proportional to  $1/r$  due to Coulomb's law

# Link atom approach

Deficiency of subtractive coupling – example

■ Reaction



■ Asymptotic behavior of total energy:

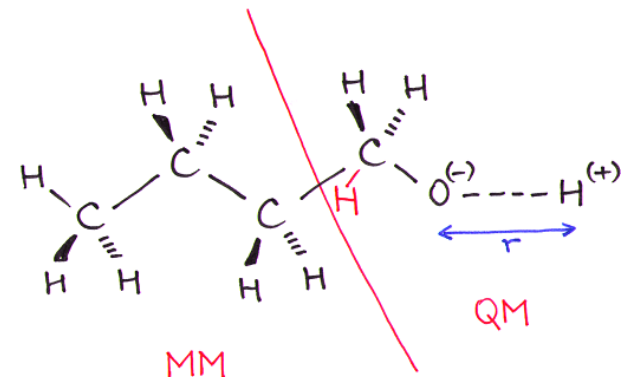
$$\lim_{r \rightarrow \infty} E_{\text{butanol}}^{\text{QM/MM}}(r) = \lim \left( -\frac{1}{r} + \frac{1}{2}k \cdot r^2 \right) = \lim r^2 = \infty$$

■ Slightly different parameter values (really possible)

- total energy grows over all limits
- absolutely **useless** calculation

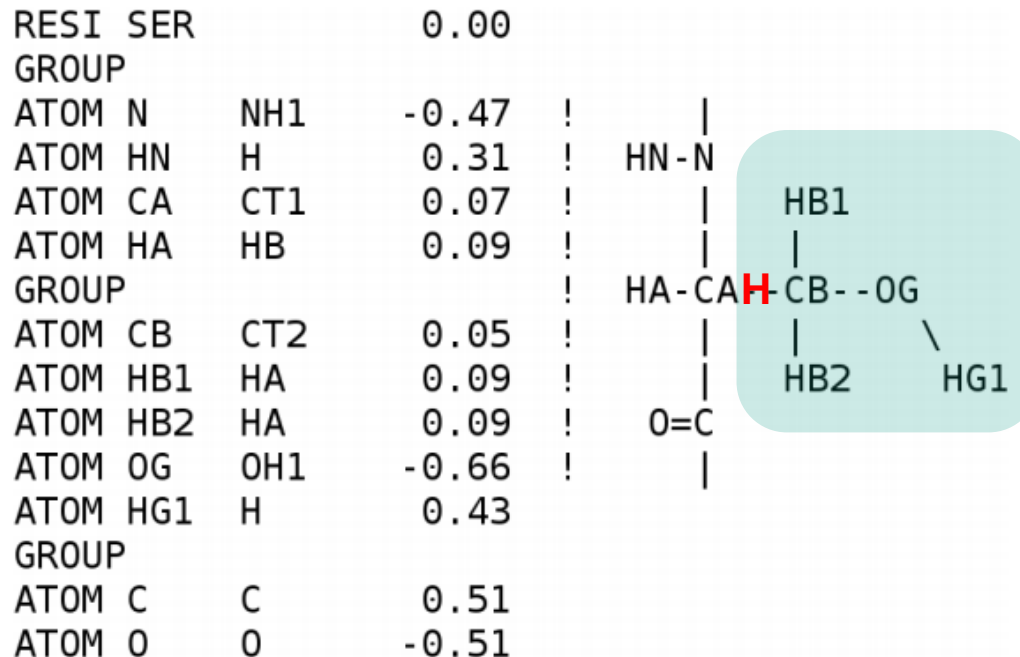
■ Additive coupling – no such problem

- methanol would be calculated with QM
- propyl would be calculated with MM
  - MM parameters for the O–H bond not required!



# Link atom approach

- Example – QM is an amino-acid side chain (starting from CB)



- CA is very close to the link atom (which is between CA and CB)
- The point charge on CA would disturb the QM region drastically
- Simply removing CA would break electroneutrality

# Link atom approach

Possible solutions to the problem:

- 'exgroup' – remove all charges of the charge group close to QM region
  - Here – CA, HA, N, H
  - Drastic – many important electrostatic interactions lost (hydrogen bonds...)
- 'div' – divide the charge of CA among the remaining atoms in the group
  - Here – q: CA = 0.07
  - After re-distribution: CA=0, HA=0.11, N=-0.44, HN=0.33
  - Strictly possible only if QM–MM boundary lies between two charge groups
  - The idea may be adapted to each specific case

# Link atoms or something else?

- Link atoms – may cause problems with non-bonded interactions
  - possibly extremely close to nearby MM atoms
  - representation of charge densities by MM point charges may lead to inaccuracy / computational instability
- Alternative – introduce no new *atoms*,  
rather treat the *orbitals* on the QM/MM boundary in a special way
- Shape of these orbitals can be held constant during the simulation  
→ ‘frozen orbitals’

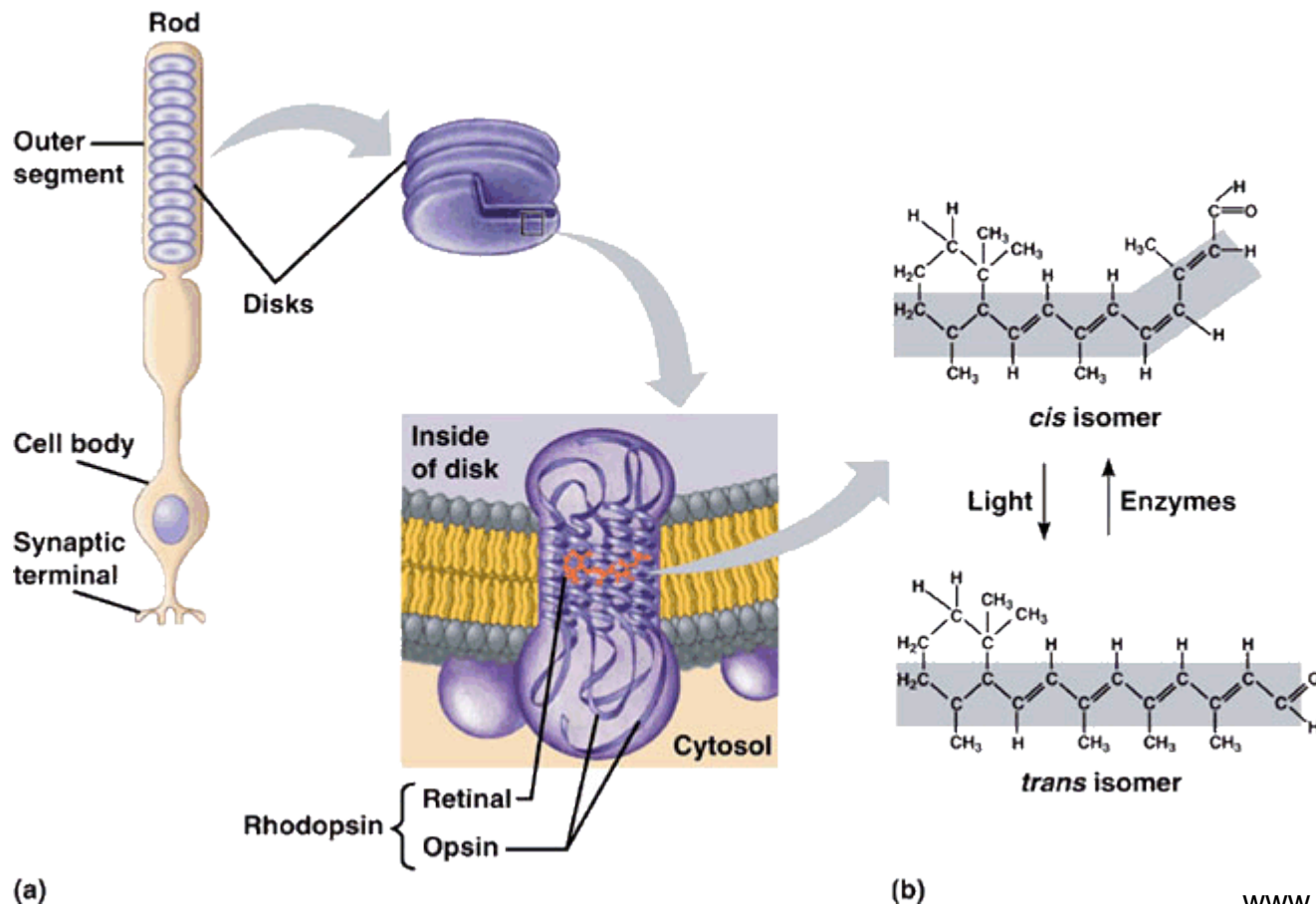
# Beyond QM/MM

- Divide the system even more – 3 disjunctive regions, QM1/QM2/MM
  - QM1 – advanced, expensive electronic structure method  
(highly correlated like CC, CAS...)
  - QM2 – more approximative and efficient electronic structure method  
(DFT or even semi-empirical)
  - MM
- 
- Example – ONIOM in the Gaussian package



# Choice of the QM region – illustration

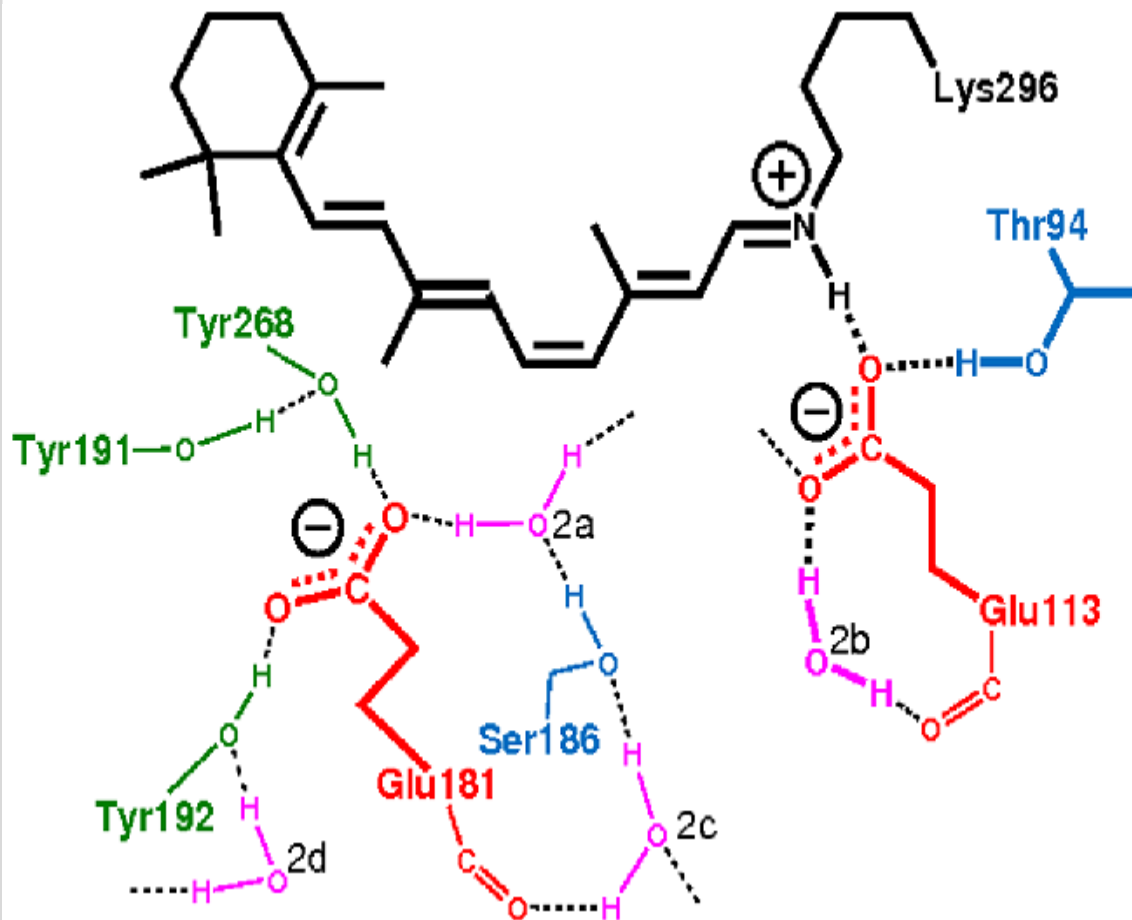
- Retinal – polyene bound covalently via a protonated Schiff base to a lysine side chain in a protein from the rhodopsin family



[www.chm.bris.ac.uk](http://www.chm.bris.ac.uk)

# Choice of the QM region – illustration

- Many different choices possible



- polyene (ring to NH):  
**bad**, boundary cuts a polar bond
- retinal + CH2  
**bad**, link atom too close to the important region
- retinal + sidechain to CB  
**fair, but** no charge transfer to Glu113 possible
- + counterion Glu113  
**better, but** no charge transfer to Wat possible
- + Wat2b+Thr94  
**good, but** no polarization at Glu181
- + Glu181  
**very good, but...**

# Adaptive QM/MM

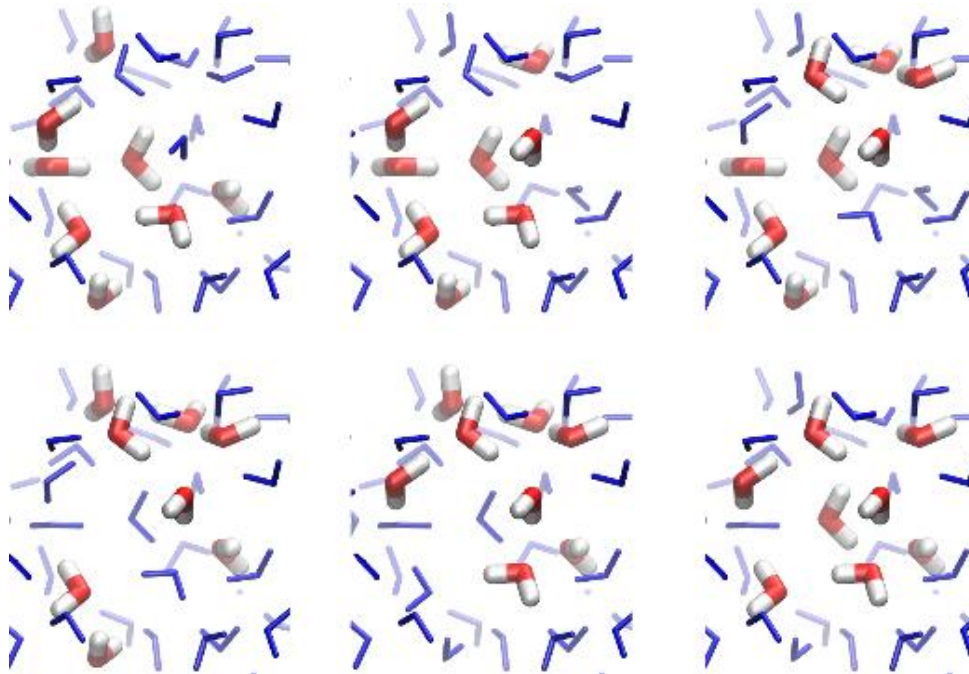
- Problematic situation – molecules entering and leaving QM zone
- Exchange of QM and MM characters desirable
- Naïve exchange impossible due to discontinuities:
  - Spatial – structure changes abruptly at the QM–MM boundary
  - Temporal – forces change abruptly upon an exchange of characters
- 1<sup>st</sup> gen. – alleviates spatial discontinuity, temporal discontinuity persists
- 2<sup>nd</sup> gen. – several partitionings, they however contain different numbers of QM molecules (QM core region, and transition region), and there is a certain required number of partitionings
- **Size-consistent multi-partitioning** adaptive QM/MM
  - removes discontinuities, avoids artificial forces
  - author: Hiroshi C. Watanabe (JCTC 10, 4242 (2014))

# Adaptive QM/MM – SCMP

- Forces obtained by averaging from several QM/MM calculations

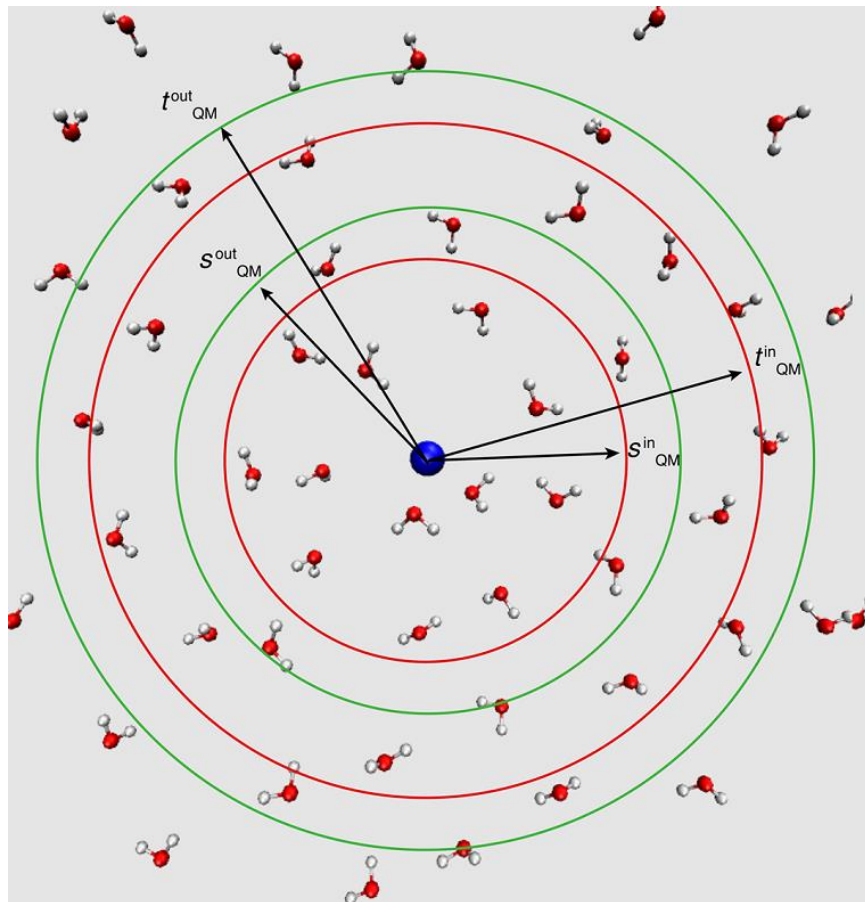
$$\mathbf{F}_j^{\text{eff}}(\mathbf{r}) = \sum_n^N \sigma^{(n)}(\mathbf{r}) \cdot \mathbf{f}_j^{(n)}(\mathbf{r})$$

- Partitionings with identical sizes of the QM region are considered



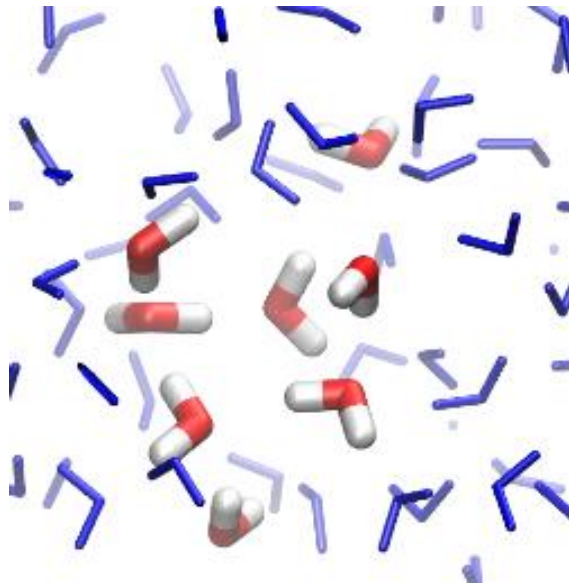
# Adaptive QM/MM – SCMP

- Weight function  $\sigma$ 
  - depends on distances of QM and MM molecules from the center



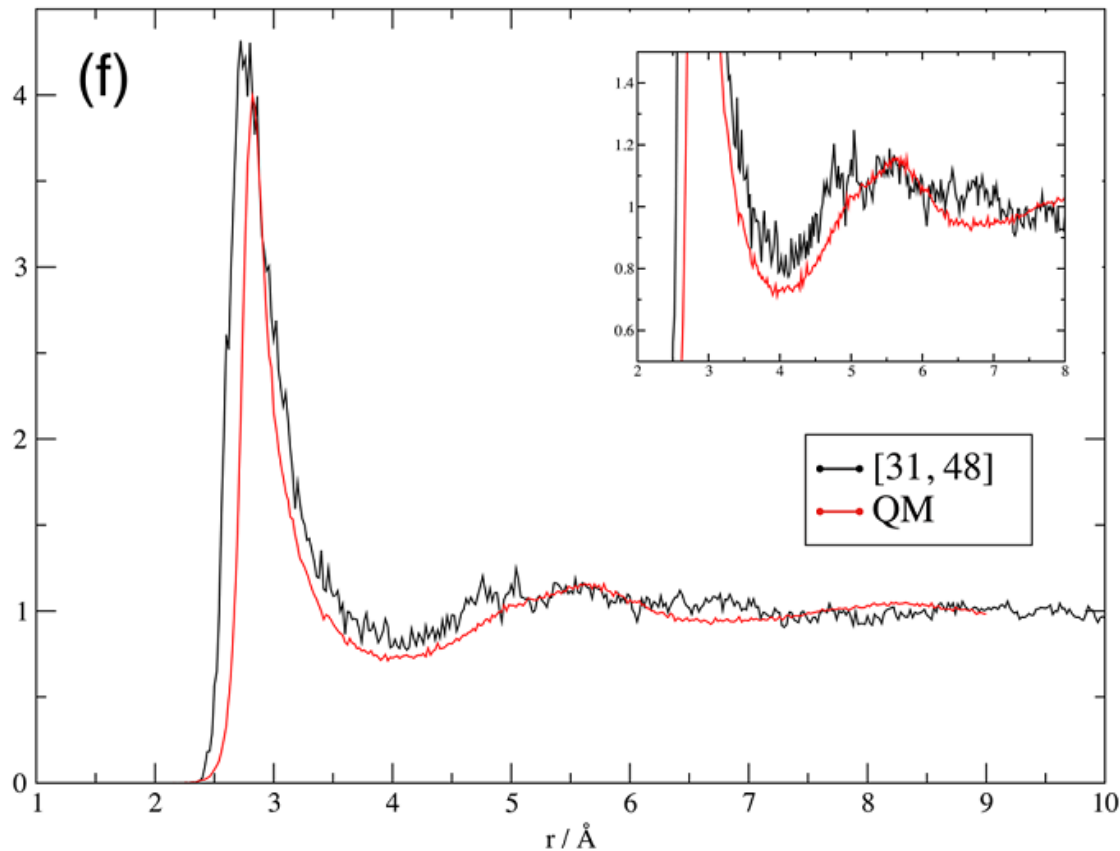
# Adaptive QM/MM – SCMP

- Weight function  $\sigma$ 
  - vanishes for a partitioning with fragmented QM zone
  - also vanishes for the most compact partitioning,
- Is  $\sigma$  of a partitioning lower than threshold?
  - replace by the most compact, so-far not considered partitioning
  - zero weight of both – discontinuities avoided



# Adaptive QM/MM – SCMP

- Example: RDF of “QM water in MM water”
  - 31 QM water molecules, 48 partitionings considered





# Adaptive QM/MM – SCMP

- Continuous forces (no artificial forces on the QM–MM boundary)
- Continuous switching of properties at the boundary
- Improvement upon many previous multi-partitioning methods
  - applicability for large QM core regions  
(no required number of partitionings)
  - conserved energy independent of the number of partitionings
- Efficient parallel implementation
  - identical sizes of QM regions – similar computational cost

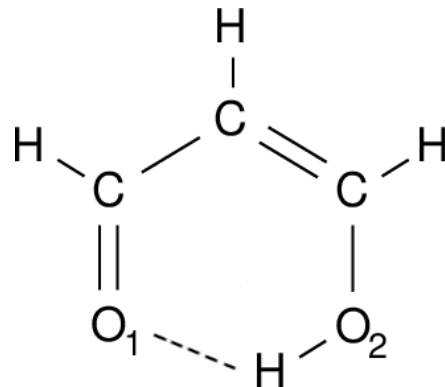
- Interface by G. Groenhof
- Communicates with external QM software  
(Gaussian, Mopac, Orca, Gamess UK)
- Additive scheme, electrostatic embedding
- DFTB3 – ‘semi-empirical density-functional theory’  
– newly implemented and fully integrated within Gromacs
- No file-based communication
- Gromacs PME routines used for electric potential – efficiency  
(PME used also for QM–QM interaction, and not full Ewald)
- Free energies – internal Gromacs functionality  
or available external tools (**PLUMED**)

; OPTIONS FOR QMMM calculations

QMMM	= yes
QMMM-grps	= MAL
QMMScheme	= normal
QMcharge	= 0
QMmult	=
MMChargeScaleFactor	= 1
QMdftbsccmode	= 3
QMdftb-telec	= 10.
QMdftb-slko-path	= /home/tkubar/DFTB/3ob/
QMdftb-slko-separator	=
QMdftb-slko-lowercase	= yes
QMdftb-slko-suffix	= -c.spl
QMdftb-partial-pme	= 1
QMdftb-dispersion	= 1

# Application 1 – Proton Transfer

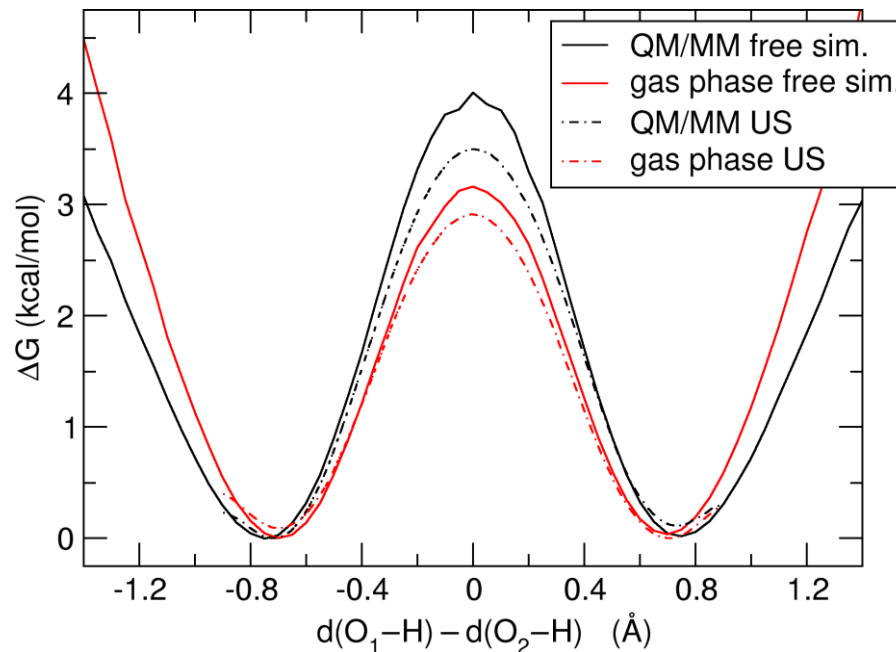
- Malonaldehyde – ultra-fast intramolecular PT



- Free energy barrier of few kcal/mol – rate of several times per ns
- Popular test case for QM/MM

# Application 1 – Proton Transfer

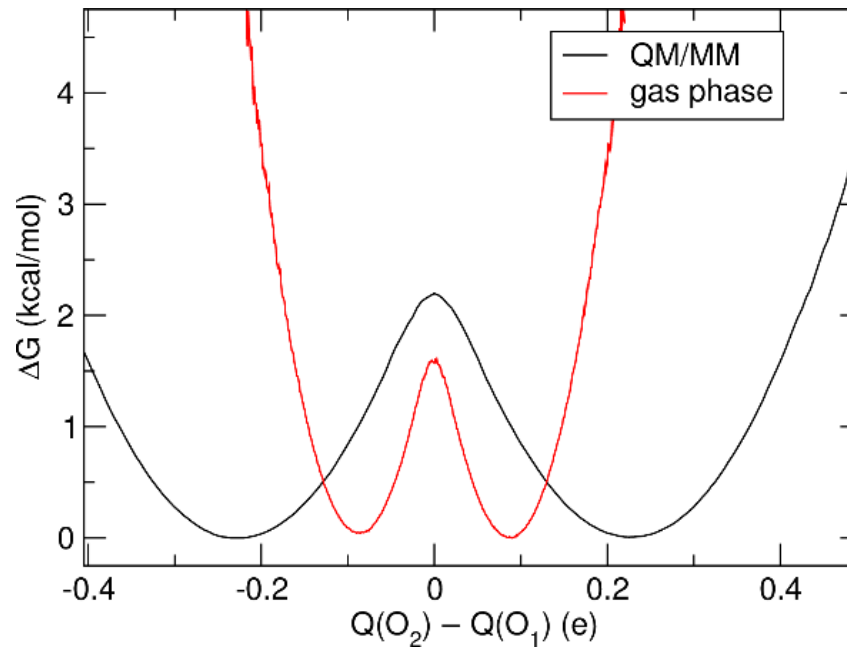
- With QM/MM and in the gas phase  
reaction coordinate: difference of O–H distances



- Barrier from free simulation slightly higher than from umbrella sampling (possibly undersampled?)

# Application 1 – Proton Transfer

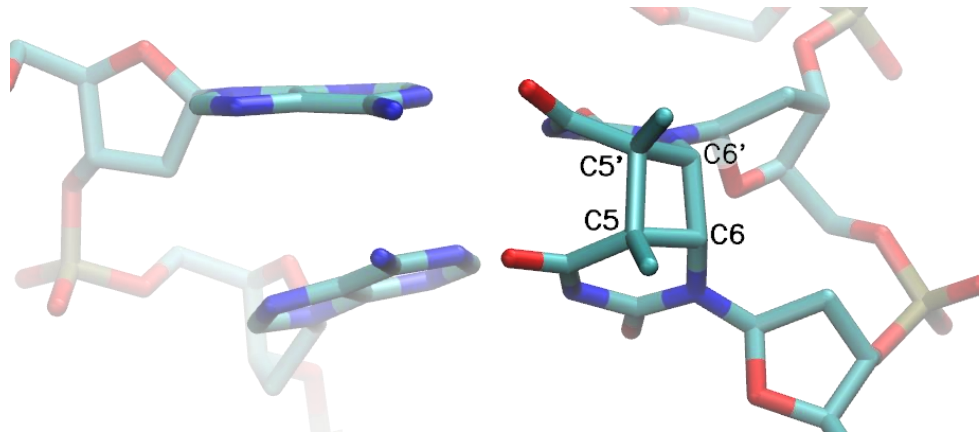
- How to choose the reaction coordinate?  
(generally non-trivial for PT with intermediate molecules)



- Perhaps non-geometrical RC – difference of Mulliken charges
- Disadvantage: biased sampling not easily possible

# Application 2 – DNA Lesion

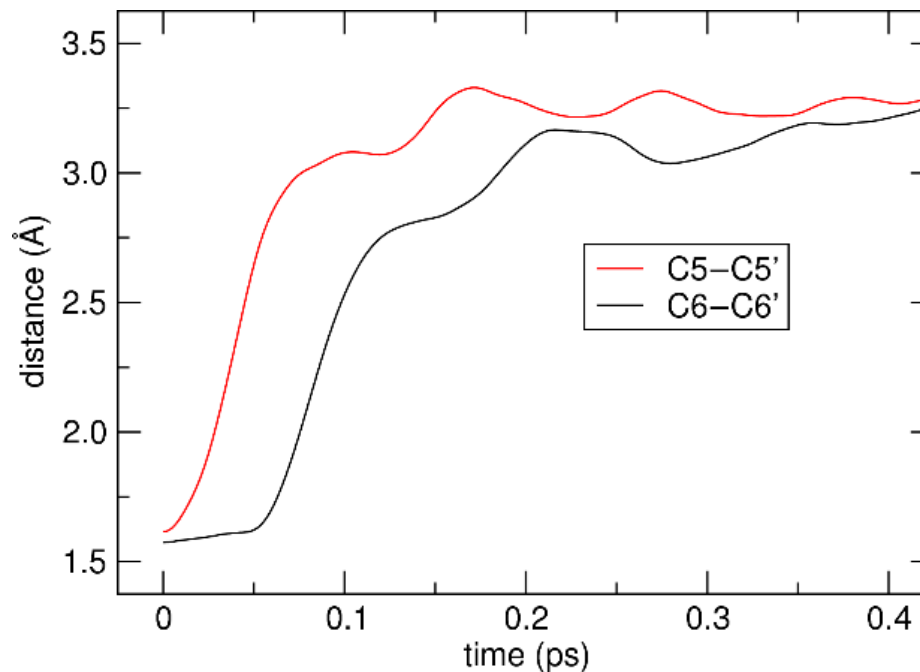
## ■ Cyclobutane pyrimidine dimer



- Stable in the electro-neutral state
- Uptake of excess electron leads to disintegration
- Mechanism debated (experimental and QM/MM studies)

## Application 2 – DNA Lesion

- Free simulation of the lesion  
after the electron uptake – radical anion

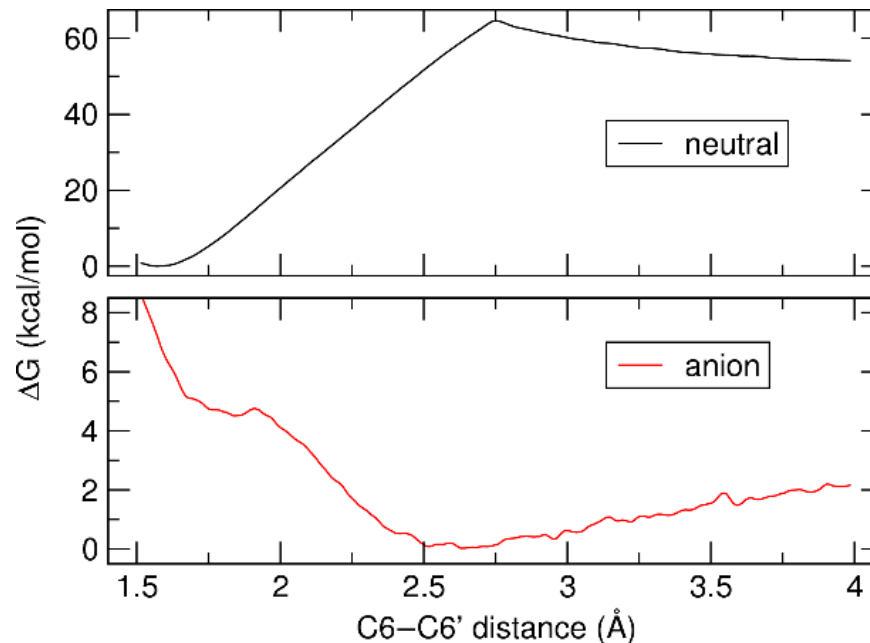


- The C5 – C5' bond breaks first, immediately
  - The C6 – C6' bond follows
- (averages from 100 simulations)



## Application 2 – DNA Lesion

- Umbrella sampling – stretching of the C6–C6' bond
  - CPD electro-neutral or after the electron uptake (radical anion)



- Meta-stable state with C5–C5' broken and C6–C6' intact?
- Tiny barrier for complete disintegration  
(NB: improved sampling and DFTB benchmark necessary)