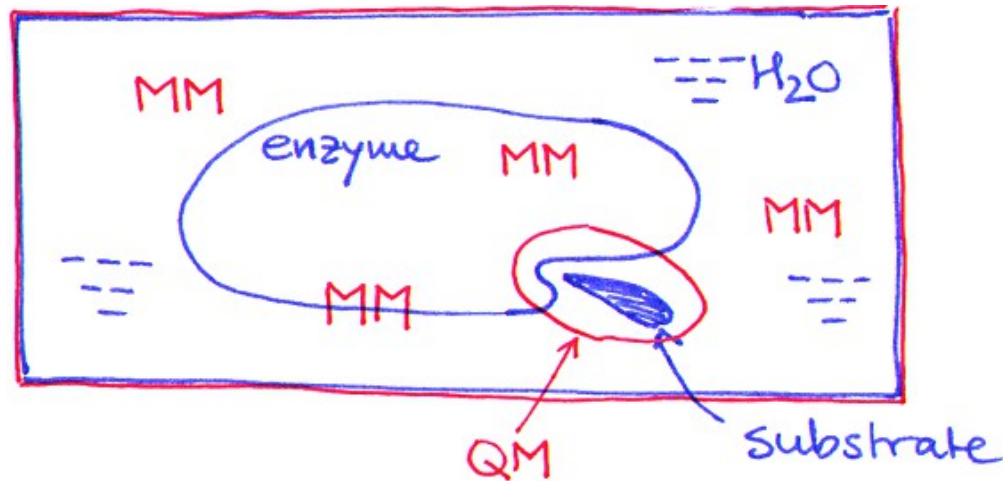


QM/MM

Marcus Elstner and Tomáš Kubař

Institute of Physical Chemistry & Center for Functional Nanostructures



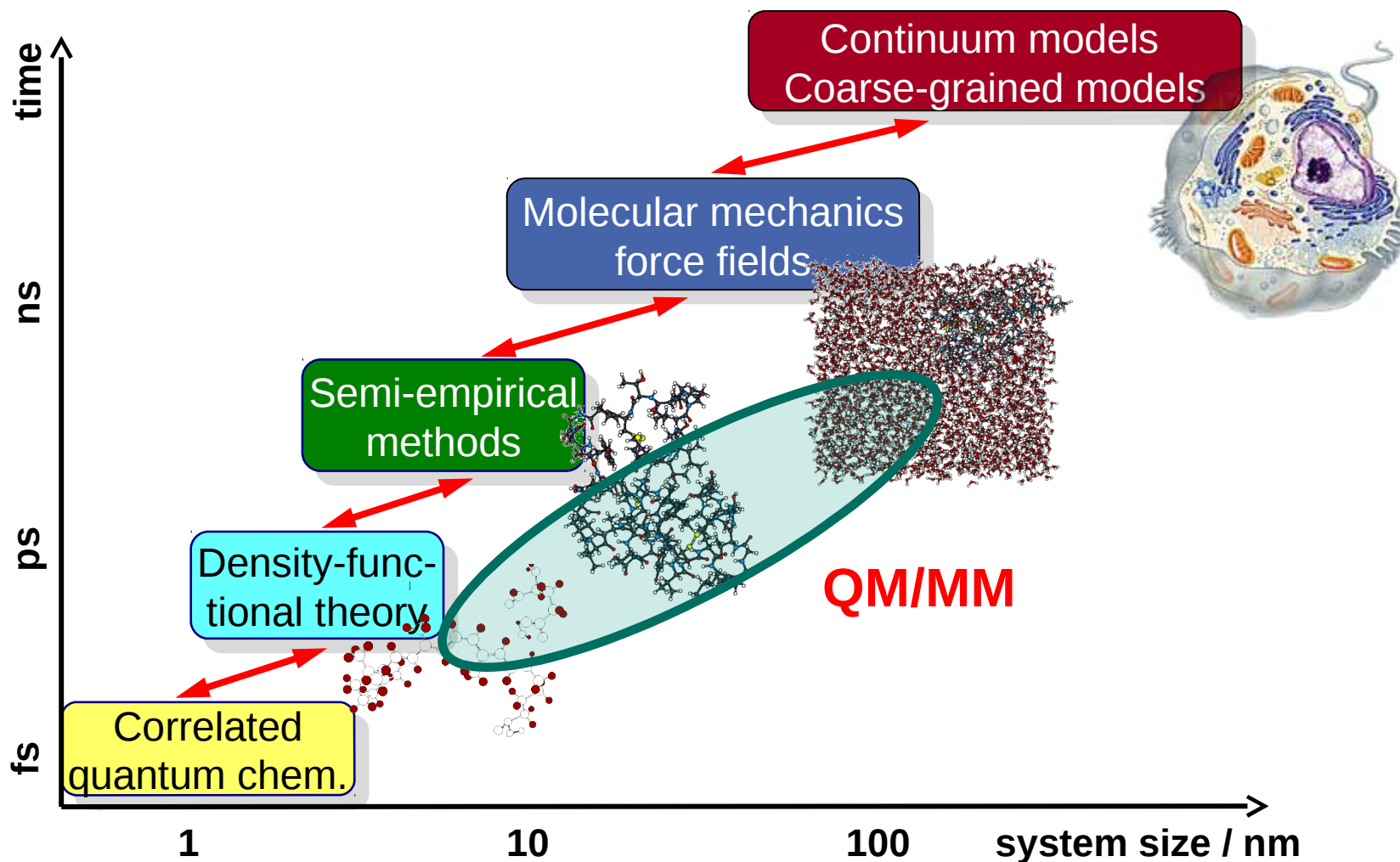
Computational chemistry

- Molecular mechanics (force fields)
 - Large efficiency
 - Crude approximations necessary
 - Constant electron density → covalent bonds cannot be created / broken
 - Chemical reactions cannot be described
 - Problems also if electron density differs for different conformations

- Quantum chemistry
 - No or little assumptions → large transferability
 - Description of chemical reactions possible
 - Larger computational cost
 - **Semi-empirical methods** – involve parameters, compromise (decreased transferability, increased efficiency)

Empirical approaches to chemical reactions

- Description of a reaction with a force field possible (but difficult)
- Using approximations that are appropriate for the specific case
- Re-parametrization of the bonds being created / broken (e.g. with Morse potential)
- Ad hoc model of the molecule, considerable parametrization effort
- Still, constant atomic charges – perhaps very problematic
- Limited or no transferability; hardly ever used (rather – QM/MM)
- Note: many chemical processes involve changes of electron density
 - Reactions involving charge transfer
 - Photochemistry



The Nobel Prize in Chemistry 2013

■ e



© Harvard University
Martin Karplus



Photo: © S. Fisch
Michael Levitt

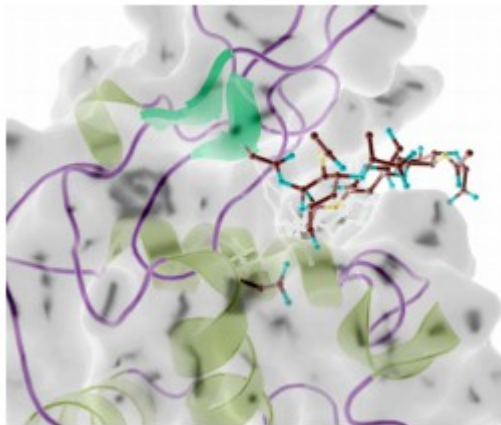


Photo: Wikimedia
Commons
Arieh Warshel

The Nobel Prize in Chemistry 2013 was awarded jointly to Martin Karplus, Michael Levitt and Arieh Warshel *"for the development of multiscale models for complex chemical systems"*.

Hybrid QM/MM methods

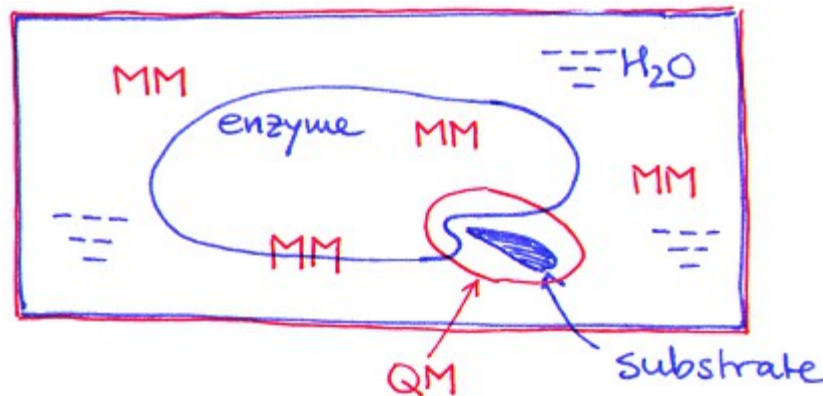
- Assumption – changing electron density localized in a small part of the molecular system
– e.g. the binding site of a ligand in an enzyme



- Only a small number of atoms are involved in a chemical reaction
- The large remainder of the system stays outside of the process, but still may interact by way of non-covalent interactions

Hybrid QM/MM methods

- Assumption – changing electron density localized in a small part of the molecular system
– e.g. the binding site of a ligand in an enzyme



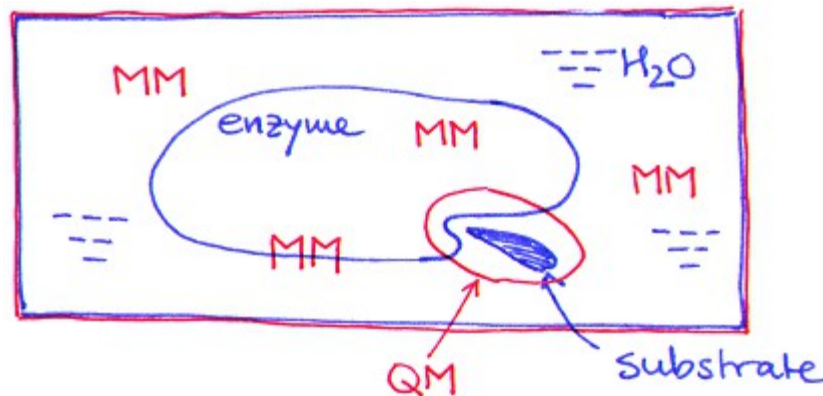
- Region where chemical reaction occurs – quantum chemistry (QM)
- The large remainder – MM force field
- Interaction of both regions must be accounted for – QM/MM / embedding

$$E_{\text{total}} = E_{\text{QM}} + E_{\text{MM}} + E_{\text{QM/MM}}$$

Hybrid QM/MM methods

- Interaction of both regions must be accounted for – QM/MM / embedding

$$E_{\text{total}} = E_{\text{QM}} + E_{\text{MM}} + E_{\text{QM/MM}}$$



- The QM and MM calculations – performed in the usual way
- Key issue – how to couple the QM and MM regions?

Goal – to obtain

$$E_{\text{QM/MM}}$$

Embedding schemes

- Many various schemes proposed
- Differences – how much of the QM–MM interaction is neglected

In the order of increasing complexity / completeness:

- Mechanical embedding
- Electronic embedding
- Polarized embedding

Mechanical embedding

- Unpolarized interactions
- Entirely independent QM and MM calculations
- Interaction between the QM and MM regions – with a force field

$$E_{\text{QM/MM}} = \sum_i^{\text{QM atoms}} \sum_m^{\text{MM atoms}} \left(\frac{q_i^{\text{Mull}} \cdot q_m}{r_{im}} + 4\epsilon_{im} \left(\frac{\sigma_{im}^{12}}{r_{im}^{12}} - \frac{\sigma_{im}^6}{r_{im}^6} \right) \right)$$

- QM atoms have to be assigned Lennard-Jones parameters and their quantum-chemical charges are used (e.g. Mulliken charges)
- Specific combinations of force fields and quantum-chemical methods lead to very good results for specific classes of molecules / reactions
- Generally, care must be taken

Mechanical embedding

- Problem – the electron density in the QM region is unaffected by ('does not feel') the MM region – therefore 'unpolarized' scheme
- As if the QM region was calculated 'in vacuum'
 - this may deviate from reality considerably
 - imagine that the reaction center of the enzyme is surrounded by charged amino acid side chains – affects the reaction strongly

Electronic embedding

- Polarized QM / unpolarized MM
- Electrostatic interaction with the MM atomic charges is included in the QM Hamiltonian
- Works with any electronic structure method
 - HF, DFT, correlated wave-function, semi-empirical...

Electronic embedding

Interaction of QM electrons with MM point charges

- moves from the QM/MM term (described with a force field) to the quantum chemical contribution
- the charge density interacts with a set of point charges
- *little* increase of the computational cost

$$\hat{H}'_{\text{QM}} = \hat{H}_{\text{QM}} - \sum_j^{\text{QM electrons}} \sum_m^{\text{MM atoms}} \frac{q_m}{r_{jm}}$$

Interaction of QM nuclei with MM point charges may remain in QM/MM

$$E'_{\text{QM/MM}} = \sum_i^{\text{QM atoms}} \sum_m^{\text{MM atoms}} \left(\frac{Z_i \cdot q_m}{r_{im}} + 4\epsilon_{im} \left(\frac{\sigma_{im}^{12}}{r_{im}^{12}} - \frac{\sigma_{im}^6}{r_{im}^6} \right) \right)$$

Electronic embedding

- A good approach with chances for acceptable accuracy
- Possible imbalance
 - QM region is polarized by MM, but MM region cannot be polarized because of the constant atomic charges

Polarized embedding

- Fully polarized interactions

- Involves a polarizable MM force field

- Every MM atom is assigned a polarizability α
- Induced point dipole on the atom obtained from the field intensity $\vec{E}$:

$$\vec{\mu}^{\text{ind}} = \alpha \cdot \vec{E}$$

- To be calculated iteratively until self-consistence
 - Possible problems – computational cost, difficult convergence of dipoles
-
- The induced dipoles interact with MM point charges as well as with the QM electron density and nuclei
 - Both QM and MM calculations are iterative
→ convergence of both is required!

Polarized embedding

- Formally the most complete approach
- Dramatically increased computational cost
- Question: Does the improvement of the quality of results justify the extreme cost of calculation?
- Electronic embedding may be sufficient in many cases

QM/MM study – work plan

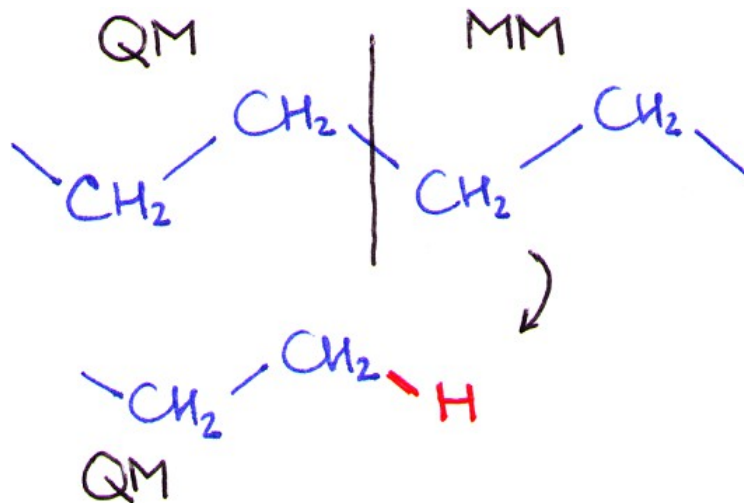
1. Define the QM region
 - shall involve all of the interactions for which MM is insufficient
2. Choose the QM (and MM) methods
 - QM is the crucial and by far most consuming part of calculation
 - accuracy as well as efficiency shall be judged
3. Determine the type of embedding (perhaps electronic)
 - also introduce LJ parameters for QM atoms (usual MM, or special)
4. Simulate
 - each MD step involves 1 QM calculation (and 1 MM, 1 QM/MM)
5. Analyze

Covalent bond crossing the QM/MM boundary

- What shall be done if a QM atom and an MM atom are bound covalently?
- Usually inevitable in studies of proteins...
- Special treatment of the boundary is necessary
(because the QM region is no real 'molecule' etc.)
- Linear combination of molecular fragments
- Link atoms (additive or subtractive coupling)
- Frozen orbitals
- other, further, current development...

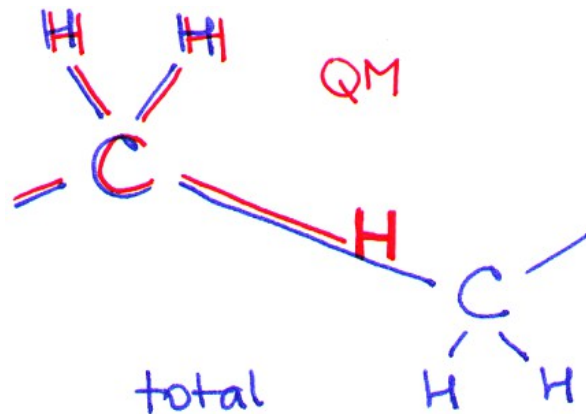
Link atom approach

- More difficult – if there is electrostatic QM–MM interaction
- Typically in proteins – polar and charged amino acids in the MM region
 - a – polarization of the electron density of the QM region
- Missing in the electronic embedding
 - description of covalent bonds crossing the QM–MM boundary
- **Link atom** – replaces the covalently bonded MM system at the boundary



Link atom approach

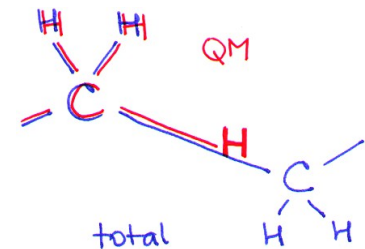
- Usually:
 - bonds between two sp³-carbon atoms chosen to be cut
 - hydrogens used as link atoms (similar electronegativity of C and H)
 - also methyl ‘atoms’ may be used with some quantum chem. Methods
- Desirable – define the QM region such that the bonds to be cut are as unpolar as possible (to minimize errors)
- Link atom – placed on the original C–C bond in a typical C–H distance



Link atom approach

Additive coupling

- Bonded interactions crossing the QM–MM boundary calculated with the MM force field
- Sometimes omitted – angles involving 2 QM atoms and 1 MM atom, dihedrals with 3 QM atoms and 1 MM atom
- MM atoms situated close to the QM system
 - almost overlap with the link atom
 - unphysically large influence on the electronic structure of the QM system
 - their charges may be scaled down or even removed
 - good alternative 1: replace point charges by gaussian charge distributions
 - good alternative 2: divide the charge among not-so-close MM atoms
(motivation: the total charge should remain, and should not move far)



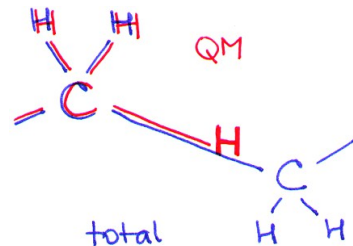
Link atom approach

Subtractive coupling

- Two MM calculations: one for the entire system, and one for the QM region including the link atom(s) – the same as for the QM calculation

$$E_{\text{total}} = E_{\text{MM}}^{\text{large}} + (E_{\text{QM}}^{\text{small}+\text{L}} - E_{\text{MM}}^{\text{small}+\text{L}})$$

(large = entire system, small+L = QM region)

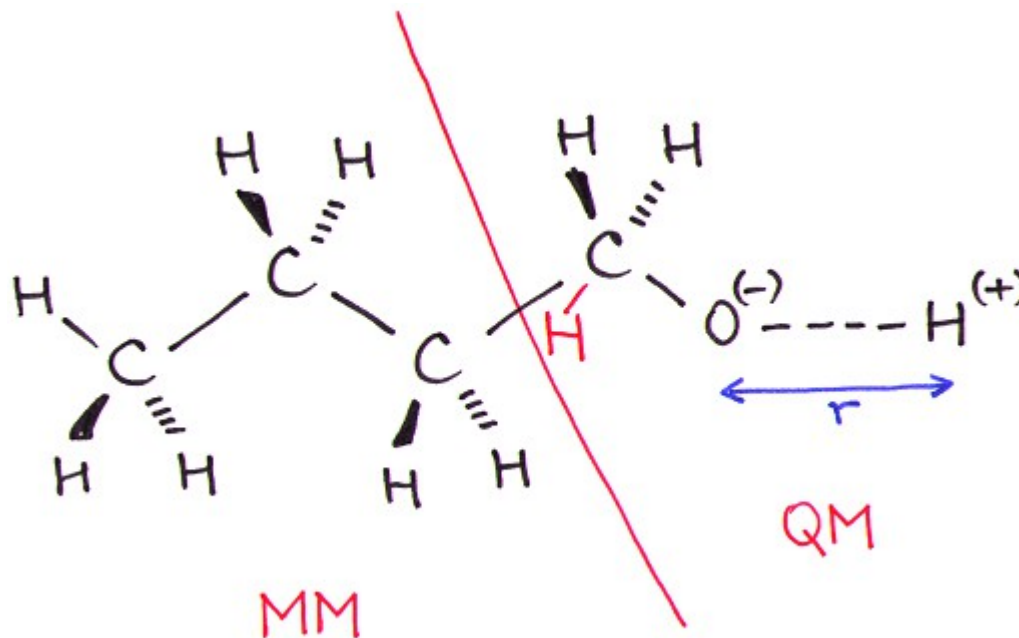


- May be deficient in some applications...

Link atom approach

Deficiency of subtractive coupling – example

- Reaction $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{OH} \rightarrow \text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{O}^- + \text{H}^+$
- QM region – methanol fragment, MM region – remainder
- Goal – evaluate the energy as a function of the O–H distance r



Link atom approach

Deficiency of subtractive coupling – example

■ Reaction



■ Assume – parameters for methanol and butanol differ only in the force constant k for the O–H bond; all remaining terms give a constant independent of r

$$\begin{aligned} E_{\text{butanol}}^{\text{QM/MM}}(r) &= E_{\text{methanol}}^{\text{QM}}(r) + \left(\frac{1}{2} k_{\text{butanol}}^{\text{OH}} \cdot (r - r_0)^2 - \frac{1}{2} k_{\text{methanol}}^{\text{OH}} \cdot (r - r_0)^2 \right) + \text{const.} \\ &= E_{\text{methanol}}^{\text{QM}}(r) + \frac{1}{2} (k_{\text{butanol}}^{\text{OH}} - k_{\text{methanol}}^{\text{OH}}) \cdot (r - r_0)^2 + \text{const.} \end{aligned}$$

■ MM energy – retains the form of r^2

■ QM energy – for large r , proportional to $1/r$ due to Coulomb's law

Link atom approach

Deficiency of subtractive coupling – example



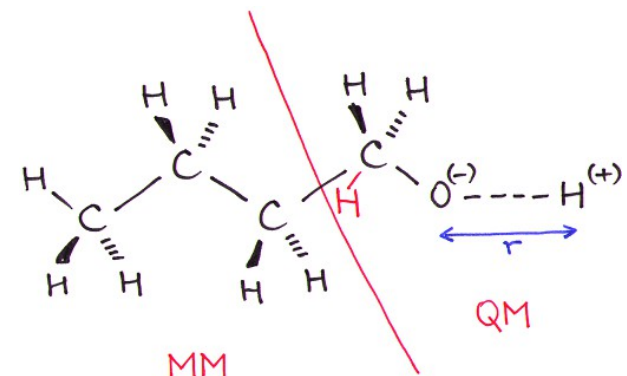
■ Asymptotic behavior of total energy:

$$\lim_{r \rightarrow \infty} E_{\text{butanol}}^{\text{QM/MM}}(r) = \lim \left(-\frac{1}{r} + \frac{1}{2}k \cdot r^2 \right) = \lim r^2 = \infty$$

- Slightly different parameter values (really possible)
 - total energy grows over all limits
 - absolutely **useless** calculation

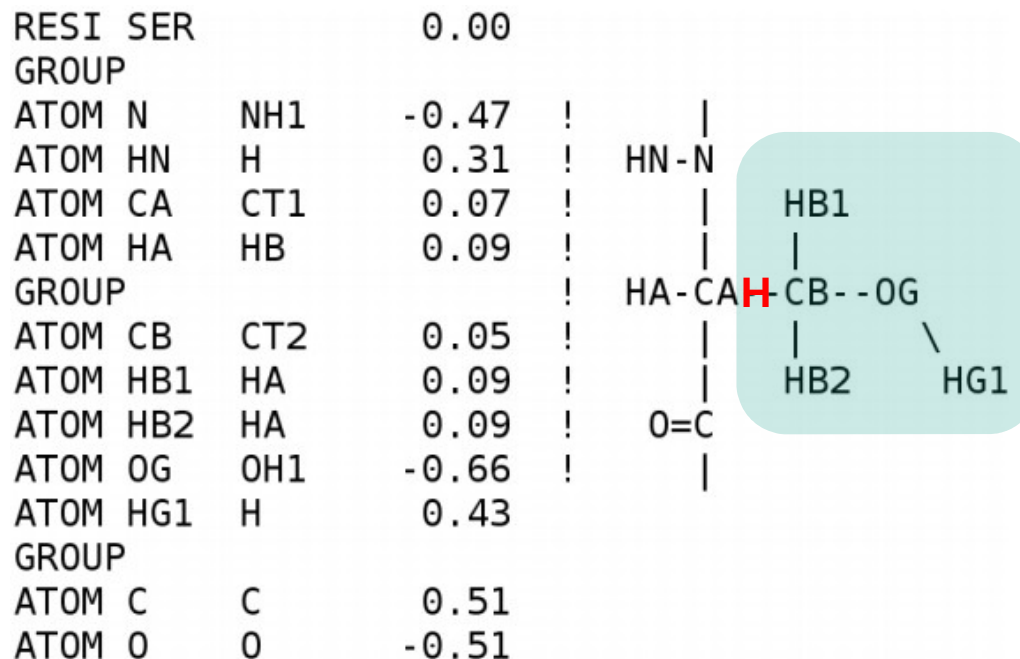
■ Additive coupling – no such problem

- methanol would be calculated with QM
- propyl would be calculated with MM
 - MM parameters for the O–H bond not required:



Link atom approach

- Example – QM is an amino-acid side chain (starting from CB)



- CA is very close to the link atom (which is between CA and CB)
- The point charge on CA would disturb the QM region drastically
- Simply removing CA would break electroneutrality

Link atom approach

Possible solutions to the problem:

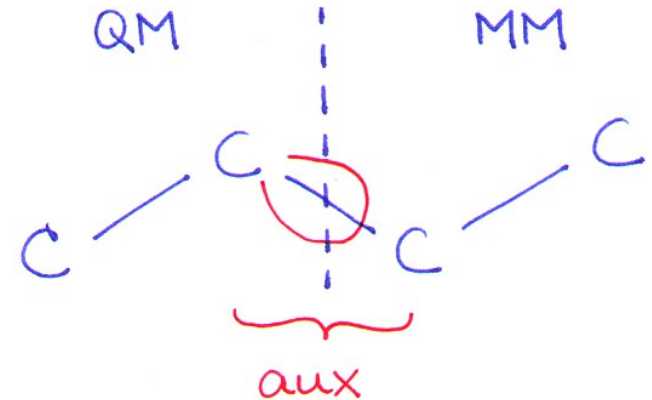
- 'exgroup' – remove all charges of the charge group close to QM region
 - Here – CA, HA, N, H
 - Drastic – many important electrostatic interactions lost (hydrogen bonds...)
- 'div' – divide the charge of CA among the remaining atoms in the group
 - Here – q: CA = 0.07
 - After re-distribution: CA=0, HA=0.11, N=-0.44, HN=0.33
 - Strictly possible only if QM-MM boundary lies between two charge groups
 - The idea may be adapted to each specific case

Link atoms or something else?

- Link atoms – may cause problems with non-bonded interactions
 - possibly extremely close to nearby MM atoms
 - representation of charge densities by MM point charges may lead to inaccuracy / computational instability
- Alternative – introduce no new *atoms*, rather treat the *orbitals* on the QM/MM boundary in a special way
- Shape of these orbitals can be held constant during the simulation
→ ‘frozen orbitals’

Frozen orbital approach

- 3 regions: QM, MM, and **auxiliary region** on the boundary
- Atoms in the auxiliary region
 - possibly a small number
 - normal nuclei
 - electron density with atomic-orbital basis
 - possess quantum character
 - their interaction with itself and with MM region can be calculated classically (interaction of frozen density with itself, of frozen density with point charges)
- Interaction of the QM region with the auxiliary
 - another term in the Hamiltonian:
 - interaction of a wave function with a frozen charge density
 - only slightly more complex than the interaction with point charges



Frozen orbital approach

■ Localized SCF method (LSCF)

- every covalent bond crossing the boundary – a single frozen orbital:
- the (hybrid) atomic orbital localized on the boundary-QM atom
- calculated once at the start of the simulation, does not change afterwards
- careful with the occupation of frozen orbitals (to handle density correctly)
 - accurate accounting required

■ Generalized hybrid orbital approach (GHO)

- the QM/MM boundary passes through an atom (instead of cutting a bond)
- the (hybrid) atomic orbitals on this atom are considered to be frozen
- populations of the GHO: resulting density equiv. to MM charge of this atom
- the remaining orbital on the atom – points inwards the QM region
 - frozen in shape; its occupation is free to vary

■ Considerable attention, ongoing development

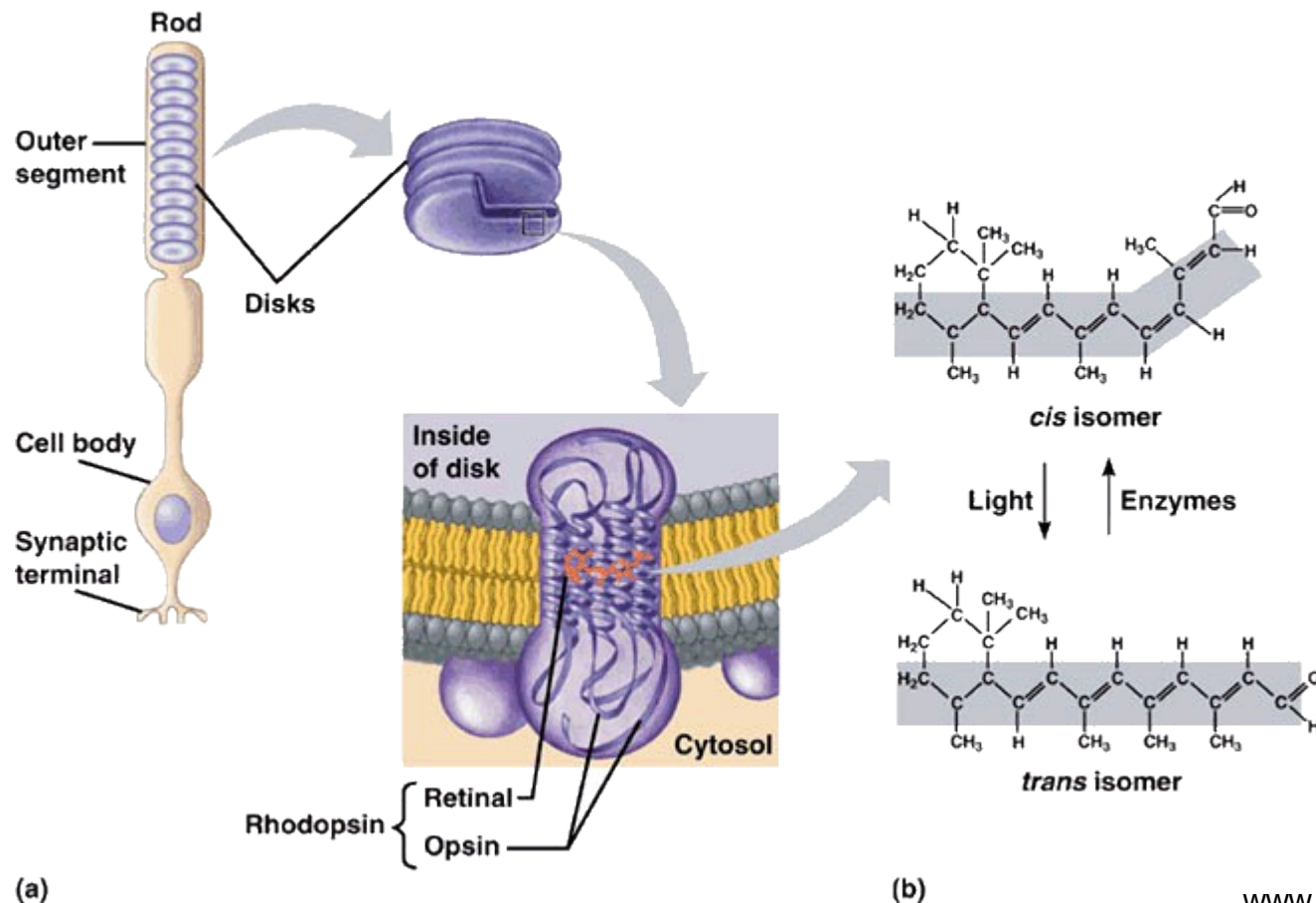
Beyond QM/MM

- Divide the system even more – 3 disjunctive regions, QM1/QM2/MM
- QM1 – advanced, expensive electronic structure method
(highly correlated like CC, CAS...)
- QM2 – more approximative and efficient electronic structure method
(DFT or even semi-empirical)
- MM

- Example – ONIOM in the Gaussian package

Choice of the QM region – illustration

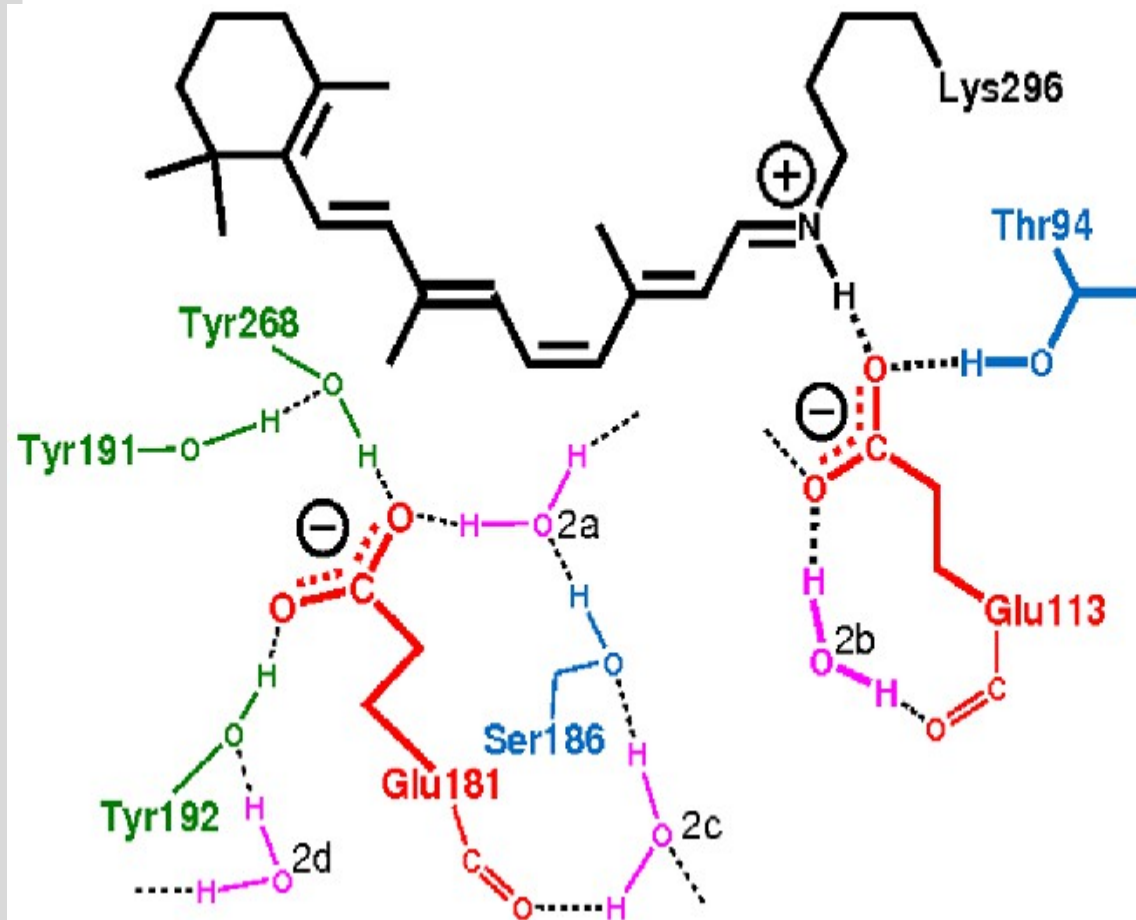
- Retinal – polyene bound covalently via a protonated Schiff base to a lysine side chain in a protein from the rhodopsin family



www.chm.bris.ac.uk

Choice of the QM region – illustration

- Many different choices possible



- polyene (ring to NH):
bad, boundary cuts a polar bond
- retinal + CH₂
bad, link atom too close to the important region
- retinal + sidechain to CB
fair, but no charge transfer to Glu113 possible
- + counterion Glu113
better, but no charge transfer to Wat possible
- + Wat2b+Thr94
good, but no polarization at Glu181
- + Glu181
very good, but...

Adaptive QM/MM

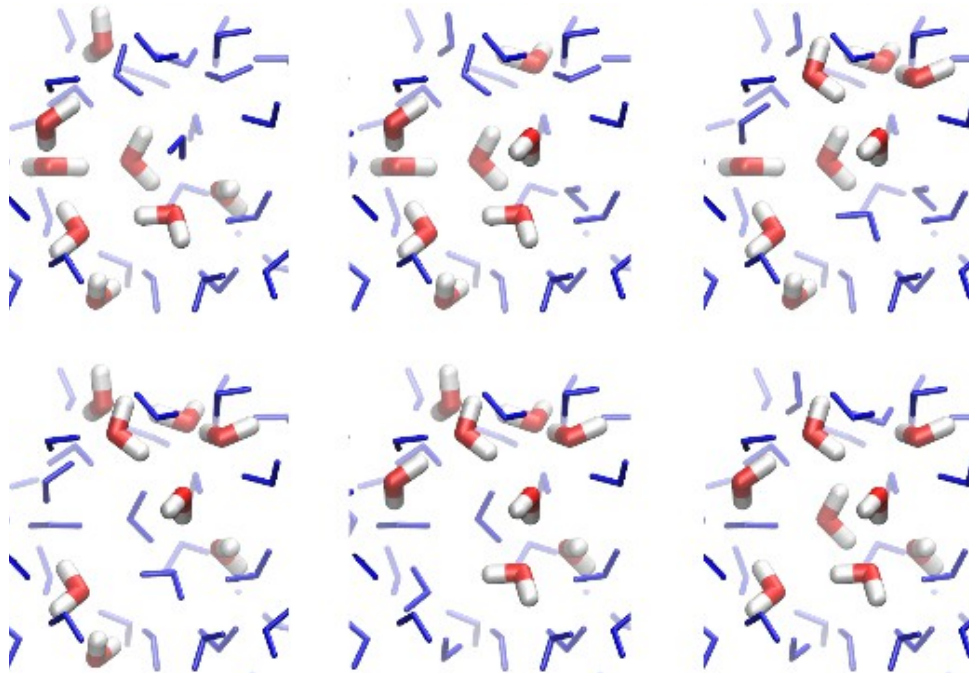
- Problematic situation – molecules entering and leaving QM zone
- Exchange of QM and MM characters desirable
- Naïve exchange impossible due to discontinuities:
 - Spatial – structure changes abruptly at the QM–MM boundary
 - Temporal – forces change abruptly upon an exchange of characters
- 1st gen. – alleviates spatial discontinuity, temporal discontinuity persists
- 2nd gen. – several partitionings, they however contain different numbers of QM molecules (QM core region, and transition region), and there is a certain required number of partitionings
- **Size-consistent multi-partitioning** adaptive QM/MM
 - removes discontinuities, avoids artificial forces
 - author: Hiroshi C. Watanabe (JCTC 10, 4242 (2014))

Adaptive QM/MM – SCMP

- Forces obtained by averaging from several QM/MM calculations

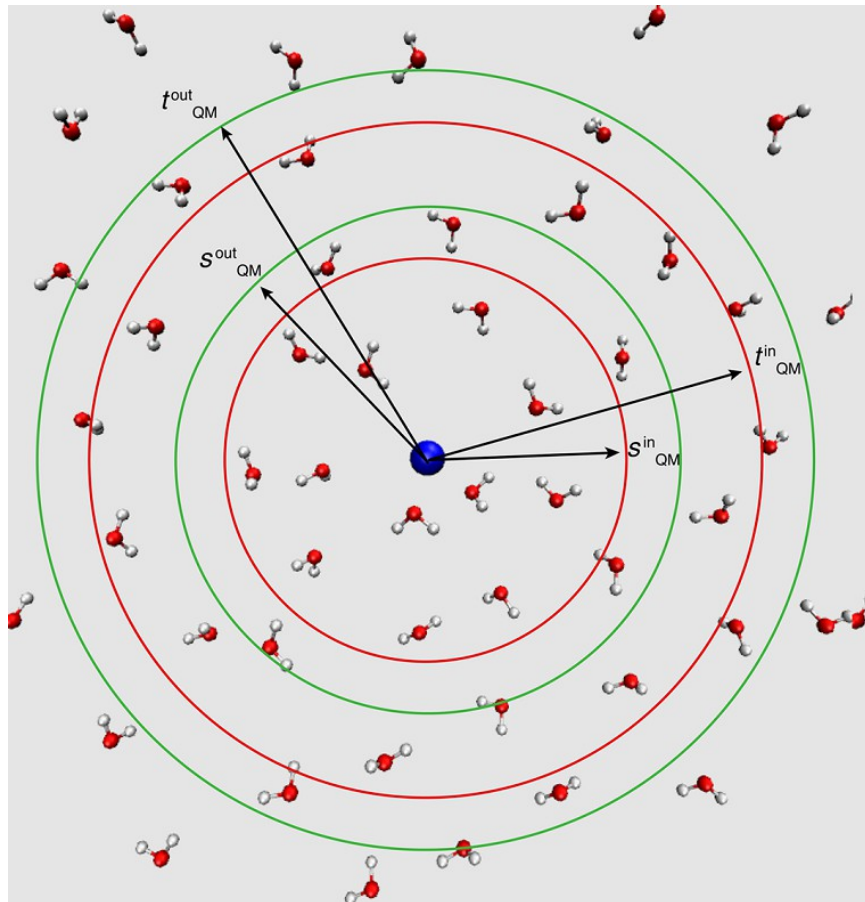
$$\mathbf{F}_j^{\text{eff}}(\mathbf{r}) = \sum_n^N \sigma^{(n)}(\mathbf{r}) \cdot \mathbf{f}_j^{(n)}(\mathbf{r})$$

- Partitionings with identical sizes of the QM region are considered



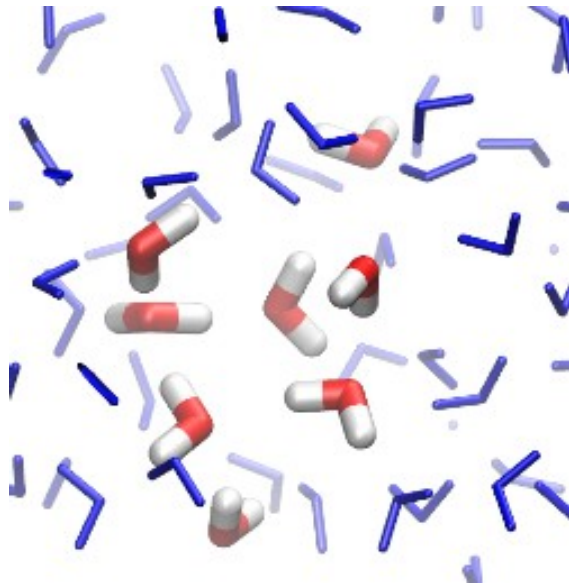
Adaptive QM/MM – SCMP

- Weight function σ
 - depends on distances of QM and MM molecules from the center



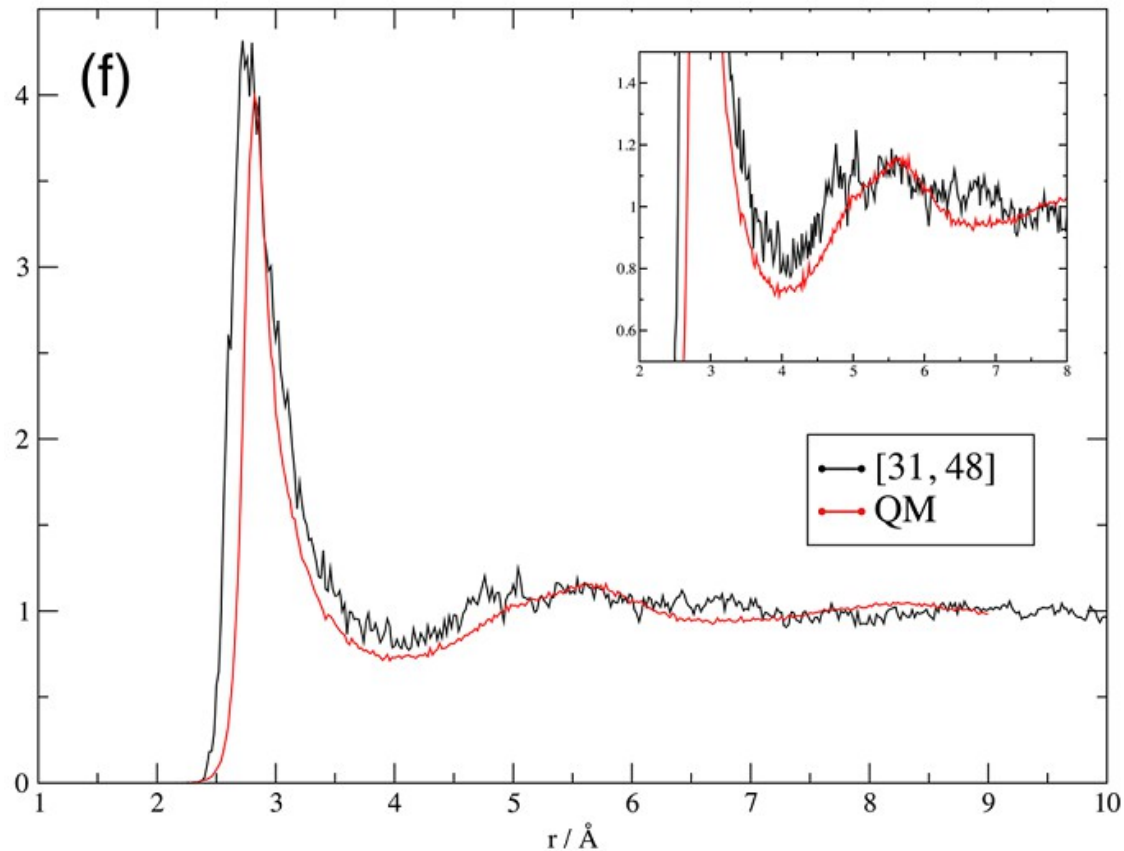
Adaptive QM/MM – SCMP

- Weight function σ
 - vanishes for a partitioning with fragmented QM zone
 - also vanishes for the most compact partitioning,
- Is σ of a partitioning lower than threshold?
 - replace by the most compact, so-far not considered partitioning
 - zero weight of both – discontinuities avoided



Adaptive QM/MM – SCMP

- Example: RDF of “QM water in MM water”
 - 31 QM water molecules, 48 partitionings considered



Adaptive QM/MM – SCMP

- Continuous forces (no artificial forces on the QM–MM boundary)
- Continuous switching of properties at the boundary
- Improvement upon many previous multi-partitioning methods
 - applicability for large QM core regions
(no required number of partitionings)
 - conserved energy independent of the number of partitionings
- Efficient parallel implementation
 - identical sizes of QM regions – similar computational cost

- Interface by G. Groenhof
- Communicates with external QM software
(Gaussian, Mopac, Orca, Gamess UK)
- Additive scheme, electrostatic embedding
- DFTB3 – ‘semi-empirical density-functional theory’
– newly implemented and fully integrated within Gromacs
- No file-based communication
- Gromacs PME routines used for electric potential – efficiency
(PME used also for QM–QM interaction, and not full Ewald)
- Free energies – internal Gromacs functionality
or available external tools ()

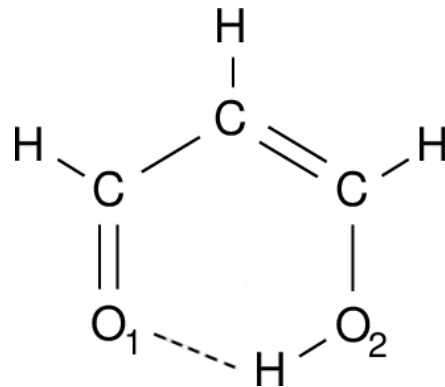
; OPTIONS FOR QMMM calculations

QMMM = yes
QMMM-grps = MAL
QMMScheme = normal
QMcharge = 0
QMmult =
MMChargeScaleFactor = 1

QMdftbsccmode = 3
QMdftb-telec = 10.
QMdftb-slko-path = /home/tkubar/DFTB/3ob/
QMdftb-slko-separator =
QMdftb-slko-lowercase = yes
QMdftb-slko-suffix = -c.spl
QMdftb-partial-pme = 1
QMdftb-dispersion = 1

Application 1 – Proton Transfer

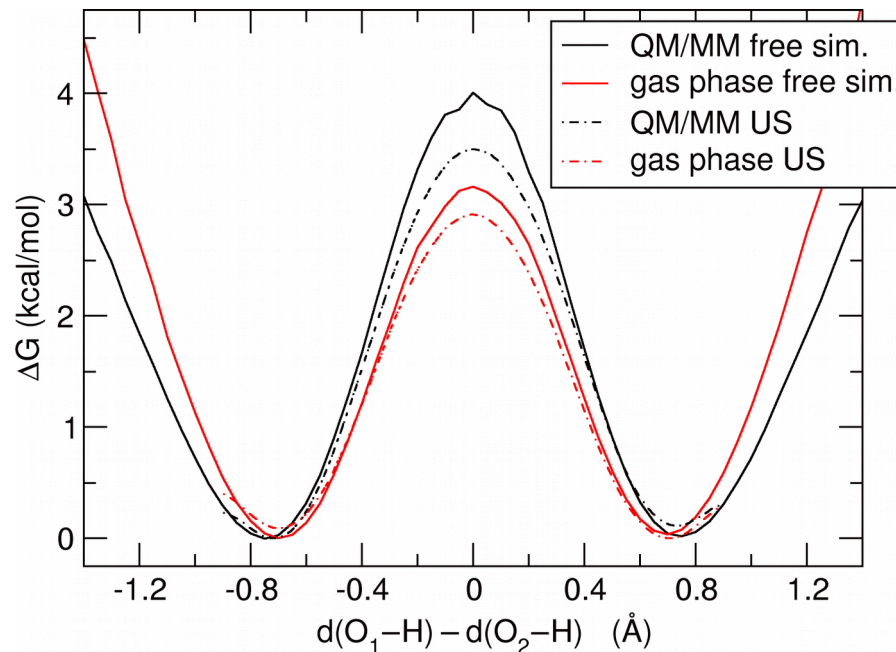
- Malonaldehyde – ultra-fast intramolecular PT



- Free energy barrier of few kcal/mol – rate of several times per ns
- Popular test case for QM/MM

Application 1 – Proton Transfer

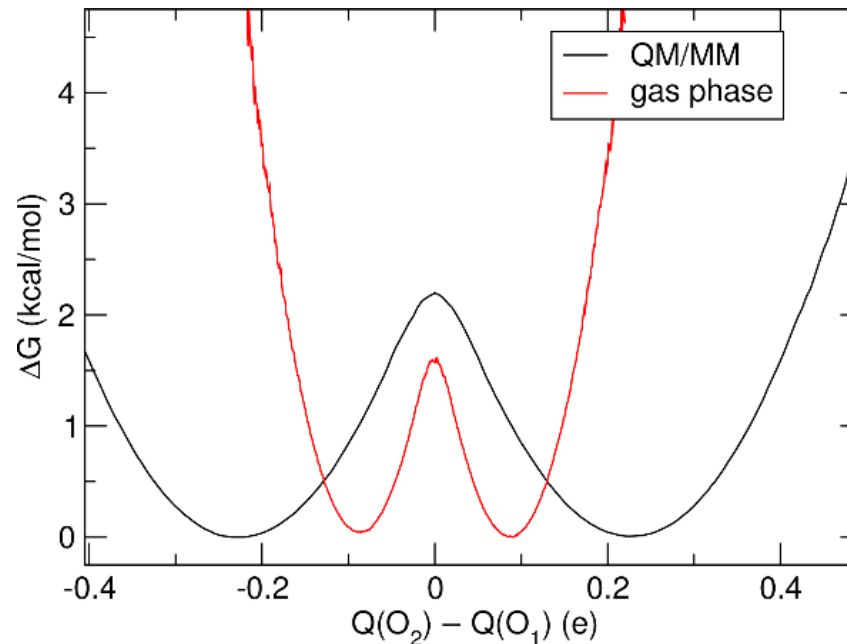
- With QM/MM and in the gas phase
reaction coordinate: difference of O–H distances



- Barrier from free simulation slightly higher than from umbrella sampling (possibly undersampled?)

Application 1 – Proton Transfer

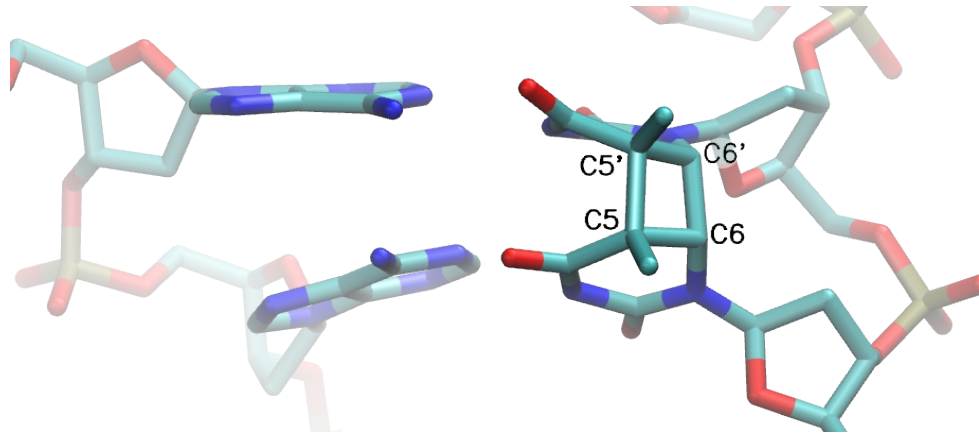
- How to choose the reaction coordinate?
(generally non-trivial for PT with intermediate molecules)



- Perhaps non-geometrical RC – difference of Mulliken charges
- Disadvantage: biased sampling not easily possible

Application 2 – DNA Lesion

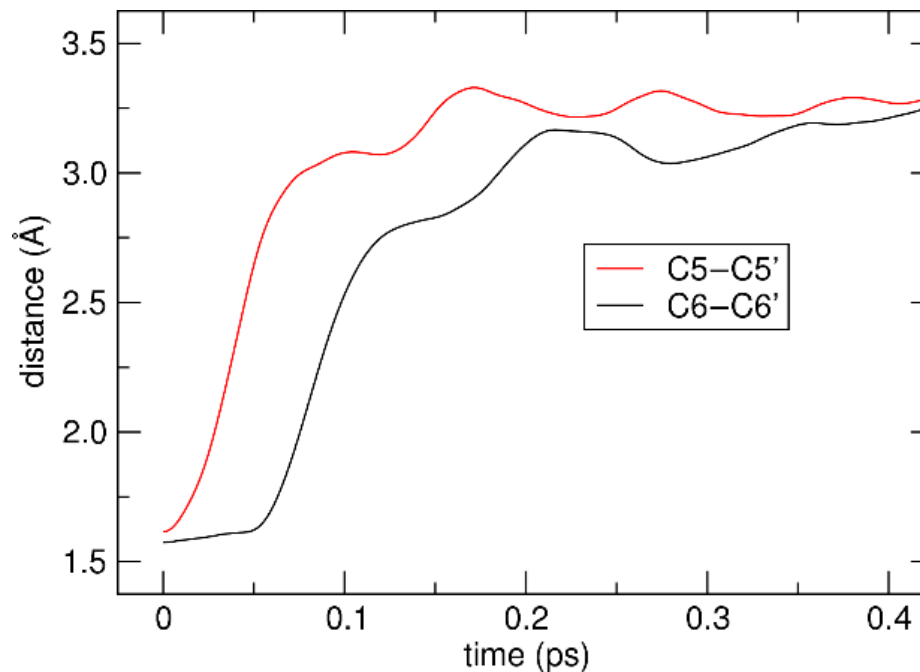
■ Cyclobutane pyrimidine dimer



- Stable in the electro-neutral state
- Uptake of excess electron leads to disintegration
- Mechanism debated (experimental and QM/MM studies)

Application 2 – DNA Lesion

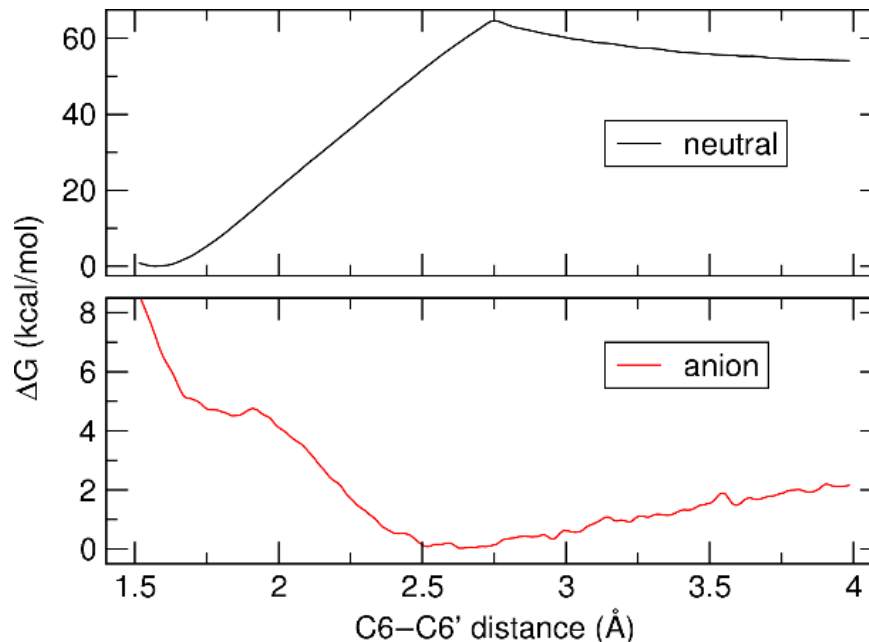
- Free simulation of the lesion
after the electron uptake – radical anion



- The C5 – C5' bond breaks first, immediately
- The C6 – C6' bond follows
(averages from 100 simulations)

Application 2 – DNA Lesion

- Umbrella sampling – stretching of the C6–C6' bond
 - CPD electro-neutral or after the electron uptake (radical anion)



- Meta-stable state with C5–C5' broken and C6–C6' intact?
- Tiny barrier for complete disintegration
(NB: improved sampling and DFTB benchmark necessary)